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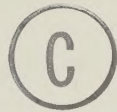


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VERBAL AND NON-VERBAL CONSERVATION AND
MATHEMATICS ACHIEVEMENT



BY
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A THESIS

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ABSTRACT

This study was motivated by the attention being given by Piaget and others to the role of conservation tasks in mental development and in the school curriculum. The purposes of the present study were: (1) to measure length conservation by using a verbal and non-verbal test of conservation, (2) to compare the verbal and non-verbal results as predictors of mathematics achievement.

One hundred grade one children from four different schools were given a mathematics achievement test and an intelligence test. One half of the sample was given a non-verbal conservation test. The other half of the sample was given a verbal conservation test. The variables of socio-economic status, age, and sex were also included in the study.

Significant correlations were found between mathematics achievement and verbal conservation scores, non-verbal conservation scores and intelligence scores.

When the effects of intelligence were controlled for there did not remain a significant correlation between mathematics achievement and non-verbal conservation. There existed a significant correlation between mathematics achievement and intelligence before and after the effects of non-verbal conservation were taken into consideration.

The major purpose of the present study was to compare the results from the verbal and non-verbal tests as predictors

of mathematics achievement. By a partial correlation procedure it was found that a significant correlation existed between mathematics achievement and verbal conservation before and after intelligence was partialled out. However, there was not a significant correlation between mathematics achievement and intelligence when verbal conservation scores were controlled for.

The above findings indicated that verbal conservation scores are superior to non-verbal and intelligence scores as predictors of mathematics achievement.

The results indicated that a non-verbal length conservation test enabled more grade one children to express their knowledge of conservation than did the verbal length conservation test.

A significant correlation existed between sex and verbal conservation. The mean score of the girls on the verbal conservation test was significantly higher than that obtained by the boys in the study.

No significant relationships existed between mathematics achievement and sex, and non-verbal conservation and sex. It was found that there was no significant correlation between mathematics achievement and socio-economic status.

The study concluded with several implications for education and some suggestions for further research.

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CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

A continuous quantity such as length or volume can only be used in reasoning if it is a permanent whole, irrespective of the possible arrangement of its parts In each and every case the conservation of something is postulated as a necessary condition for any mathematical understanding (Piaget, 1952, pp. 3-4).

Obviously, conservation, which is a necessary condition of all experience and all reasoning, by no means exhausts the representation of reality or the dynamism of the intellectual process Our contention is merely that conservation is a necessary condition for all rational activity (Piaget, 1964, p. 3).

It is obvious that Piaget attaches great importance to the acquisition of the concept of conservation. This importance is further emphasized by the findings of a number of research projects. Pflederer (1964) found that children's ability to conserve on substance and number was significantly related to their performance in some musical tasks. A similar relationship was found in art by Lansing (1966) and in reading comprehension by Rawson (1965). Almy (1966) found that the ability to conserve was related to general school achievement.

Various methods have been used to measure or gauge conservation in young children. The impetus has been provided by Piaget who has done extensive study by using verbal methods in his experiments. He has found that 75 per cent of his subjects between seven and eight years of age can be

classified as conservers of length. Elkind (1961), Lovell et al. (1962), Murray (1965) and Smedslund (1963) have reported similar findings.

Braine (1959) and Sawada (1966), who attempted non-verbal methods of measuring length conservation, found the age level of conservation to be between four and six years.

Using verbal conservation tests based mainly on Piaget's work, studies (Dodwell, 1961; Hood, 1962; Overholt, 1964; Almy, 1966; Steffe, 1966; Reimer, 1968; Cathcart, 1969) have investigated the relationship between various aspects of conservation and achievement in mathematics. These studies have, with the exception of Overholt, found a significant and positive relationship between mathematics achievement and conservation of substance.

There remains little doubt that conservation is related to school achievement in general and to mathematics achievement in particular. However, even if a child is classified as a non-conserver by a verbal length conservation test this may be inadequate evidence that the child has not developed the concept of length. Perhaps a non-verbal test would have yielded different results. The findings mentioned earlier indicate that the results may be affected by the degree of verbalization in testing procedures used in measuring conservation. Braine, in criticizing Piaget, states:

It would seem to be intrinsically impossible to study how a concept develops with methods which employ verbal cues to evoke the concept. For if the child understands the verbal cue, he must already have the concept (1962, p. 46).

I. PURPOSE OF THE STUDY

The purpose of this study was two-fold:

(1) To measure length conservation in grade one children by using a verbal and non-verbal test of conservation. Previous researchers have investigated length conservation in either a verbal or non-verbal way, but have not attempted to compare the results of both methods as measuring devices of acquisition of the concept of length. It was hoped that this investigation would provide information regarding those measuring techniques.

(2) To compare the verbal and non-verbal results as predictors of mathematics achievement. Research has provided ample evidence that there is a significant relationship between conservation and achievement in mathematics. There has not been much research devoted to the relative efficacy of the results of different methods of measuring conservation as predictors of mathematics achievement.

II. MAJOR HYPOTHESES

- (1) There is no significant relationship between the ability to conserve length as measured by:
- (a) a non-verbal conservation test and

- achievement in mathematics;
 - (b) a verbal conservation test and achievement in mathematics;
 - (c) a non-verbal conservation test and achievement in mathematics, controlling for the effects of intelligence;
 - (d) a verbal conservation test and achievement in mathematics, controlling for the effects of intelligence.
- (2) There is no significant relationship between:
- (a) mathematics achievement and sex;
 - (b) ability to conserve length as measured by a non-verbal test and sex;
 - (c) ability to conserve length as measured by a verbal test and sex;
 - (d) mathematics achievement and socio-economic status.
- (3) There is no significant relationship between:
- (a) the ability to conserve length as measured by a non-verbal conservation test and intelligence;
 - (b) the ability to conserve length as measured by a verbal conservation test and intelligence;
 - (c) achievement in mathematics and intelligence, controlling for the effects of non-verbal

conservation scores;

- (d) achievement in mathematics and intelligence, controlling for the effects of verbal conservation scores.

- (4) There is no significant difference between the mean score on the non-verbal length conservation test and on the verbal length conservation test.

III. DEFINITIONS

Mathematics Achievement. The subject's scores on the achievement test designed for this study.

Intelligence. The subject's score on the Lorge-Thorndike, Intelligence Test, Level I.

State. A condition or configuration of an object exhibited as a certain magnitude of length.

Transformation. An operation of changing an object or objects from one state into another state.

Non-Invariant Transformation. Any transformation under which a given property does not remain invariant. In this study a non-invariant transformation is any transformation which changes the magnitude of the length of an object.

Invariant Transformation. Any transformation under which a given property of an object or objects remains invariant. It is a transformation which conserves the physical property or properties being studied.

Conserver. Any subject who responded correctly to

seventeen or more items out of a total of twenty-four items on the conservation tests. (Using the binomial distribution as a model this is significantly better than chance at the .01 level.)

Rotation. A transformation involving the angular displacement of an object about a self-contained origin.

Translation. A transformation involving the linear displacement of an object without rotation.

Transitivity. Refers to the logical inference that if A is greater than B and B is greater than C, then A is greater than C. In this study A, B, and C will refer to the lengths of the objects used in the conservation tests.

IV. LIMITATIONS

In interpreting the data of this study the following limitations should be borne in mind.

(1) The sample for the study was selected from a population of grade one children in the Edmonton Public School System, Alberta. It was assumed that this sample was representative of urban children around the age of six years six months.

(2) Any attempt to compare methods of measuring the acquisition of the length concept in children beyond the grade one or grade two level would be futile since they would likely be conservers on both the verbal and non-verbal methods used in this study. If a relationship between

achievement and conservation tests is sought in grades beyond grade one, then conservation tasks involving activities other than the length dimension should be used. Hence, any relationship which was found between achievement and length conservation will apply at most to the particular age group of the study.

(3) The instrument used to test mathematics achievement was constructed by the investigator. The validity of the test is unknown but efforts were made to ensure that the test measured those concepts for which it was intended.

(4) In the verbal conservation test the wording of the questions may have varied slightly in reaction to the subjects' responses. This was controlled as much as possible.

V. SIGNIFICANCE

It was indicated in the introduction that conservation and achievement are important factors in a school education. Although there are degrees of differences in the findings of many studies that have been done, it appears, as Almy (1966) suggests, that this area is deserving of even greater emphasis than it has received in the past. The method of measuring concepts must be further investigated. Braine (1959) and Sawada (1966) suggest the importance of measuring techniques.

This study was concerned with the methods of measuring the concept of length and its relationship to mathematics achievement. A knowledge of this relationship should be of

some assistance to teachers in diagnosing the stage of development which the child has reached and thereby help in suggesting the type of activities in which he might more beneficially engage.

Reimer (1968) used various types of conservation tasks -- number, quantity, and length. He found no significant difference between the various types of conservation as predictors of mathematics achievement. Only length conservation was used in the present study. However, Reimer's finding makes it possible to generalize the findings of this study to include all types of conservation tasks in which grade one children engage.

The significance of this study may be summarized as follows:

(1) Conservation plays a very important role in education. It is, therefore, necessary that a technique be available to measure concepts in young children at as early an age as the concept is present.

(2) The relationship between verbal and non-verbal methods as measuring techniques and their relative efficacy as predictors of mathematics achievement need to be further investigated.

(3) That there are specific stages in child development in which conservation of various concepts take place has obvious implications for curriculum sequence. For example, instruction in the length concept before the child

has acquired conservation of that concept, may cause the child to "acquire a verbal fluency which masks a conceptual defect" (Murray, 1965, p. 65).

VI. THE EXPERIMENTAL SETTING

The following is an overview of the experimental design. A more detailed account is reported in Chapter III.

The population from which the sample was drawn consisted of the total grade one enrollments of four elementary schools of the Edmonton Public School System. The sample consisted of one hundred grade one children selected from this population. The sample was randomly divided into two groups.

A non-verbal test of conservation of length, constructed by Sawada (1966) and revised by the investigator after a pilot study, was administered to each subject in one group on an individual basis. This followed a training session to permit subjects to become familiar with the apparatus used in the non-verbal test. Following a short training session, a verbal test of conservation of length, was administered individually to each subject in the other group. The items on this test corresponded to those on the non-verbal test. The main difference in the two conservation tests was that the non-verbal test used a three-way response apparatus and calipers, whereas the verbal test used verbal instructions and required verbal responses by the subjects.

In both tests all items were designed to measure corresponding mathematical concepts. The total sample was administered an achievement test, constructed by the investigator (Appendix A), and the Lorge-Thorndike Intelligence Test, Level I.

The main analysis consisted of an examination of the two methods, verbal and non-verbal, as gauging dimensions of length conservation, and a comparison of the two methods as predictors of mathematics achievement.

VII. OUTLINE OF REPORT

A review of the relevant literature will be presented in Chapter II. Chapter III contains a detailed account of experimental design, testing procedures, and the research procedures used to test the hypotheses. The results of the data analysis is contained in Chapter IV. The final chapter, Chapter V, includes a summary and discussion of the findings, and contains some implications for education and further research.

CHAPTER II

REVIEW OF RELATED LITERATURE

The purpose of this chapter is to discuss briefly Piaget's experiments on length conservation and to summarize other relevant studies in this area. In addition, investigations into the relationship between conservation and mathematical achievement are summarized and discussed.

Various conservation tasks have been extensively observed by Piaget and his collaborators at Geneva. A most detailed account of the conservation of length is reported in Piaget and Inhelder (1950).

Other researchers (Almy, 1966; Murray, 1965; Lovell et al., 1962; Glick and Wapner, 1968; Elkind, 1961; Delacy, 1967) have attempted replication of Piaget's experiments using verbal tests similar to Piaget's.

A number of researchers have investigated conservation of length in a less verbal way than Piaget. Some of these are Wallace, 1967; Wohlwill, 1968; Smedslund, 1961, 1963; Silverman et al., 1968; Murray, 1968, 1970; Braine, 1959; and Sawada, 1966.

I. TECHNIQUES OF MEASURING LENGTH CONSERVATION

Verbal

The main concern of Piaget (1960) was to assess the child's ability to conserve, measure, and otherwise take into

account entities like distance, length, area and volume. As an overview of Piaget's study into the concept of length, two experiments are commented upon briefly.

In one experiment a child was presented with a straight stick and an undulating strip of plasticine. The stick and plasticine, whose end points were coterminous, were placed a few millimeters apart. The child was asked questions as to the relative lengths of the objects before and after various transformations. Approximately 80 per cent of the children four and one-half years and younger gave incorrect replies, whereas only 10 per cent of those older than five and one-half gave incorrect replies. Piaget concludes that the non-conservers estimated length only in terms of its end-points. The conservers, however, were aware of the intervals or segments which lie between the extremities.

In another experiment, subjects were shown two sticks of identical length with their extremities coterminous. All agreed that the sticks were of equal length. One of the sticks was moved forward one or two centimeters and the child was asked if they were still the same length. The non-conserver maintained that the stick which was moved forward was longer, indicating that he was thinking only in terms of the further extremities. The responses given by others indicated that they were in a transitional stage and still others regarded conservation as a natural and logical

necessity.

Piaget asserts that failure to conserve length is often connected with a lack of attention to empty sites and change of spatial position. In examples dealing with privileged segments and the number of elements, failure to conserve may arise from faulty subdivision. In such cases overall length is determined by the length of a single segment or by the number of segments in the line.

Not until a child's 'schemata' is structured so as to overcome these deficiencies will he be a conserver of length. Piaget's findings suggest this stage to be around seven years of age for the subjects in his study.

In every case, subdivision on the one hand and order and change of position on the other are not synthesized and are relatively undifferentiated. It is this lack of coordination between its two fundamental aspects which accounts for the non-conservation of a qualitative linear series (Piaget, 1960, p. 112).

The following points are noted in connection with Piaget's work on length conservation.

(1) The account of Piaget's observations is highly theoretical with no indication of the differences among his subjects other than age. Piaget (1964) has noted that age does not form a rigid division of various stages of development. "It is relative to the society in which one is working" (p. 31). If Piaget had given more information on his subjects, for example, I.Q., it may have been possible to make further generalizations from his studies.

(2) The highly verbal nature of Piaget's interviews with his subjects makes it difficult to determine the cues to which the children responded when they reacted to his questions. It is uncertain whether the subjects responded to an adult meaning of words such as 'longer', 'shorter', or the meaning of the words as understood by children of that age. It may be that the medium of communication was inappropriate.

Many researchers, mostly psychologists, have replicated Piaget's studies of conservation and have subjected their results to analysis. It appears that they are convinced of the importance of conservation to education and its implications for child development.

Some investigators have tried to approximate Piaget's method while others have applied new testing methods as a gauge for measuring the acquisition of length conservation.

David Elkind, a noted American psychologist, has done extensive work in systematic replications of experiments originally performed by Jean Piaget.

The purpose of Elkind's (1961a) study was to replicate Piaget's experiments on children's understanding of length concepts and other forms of metrical geometry. In another study (Elkind, 1961b), one hundred and seventy-five children were asked to predict, judge and explain the conservation of mass, weight and volume in a testing procedure resembling that of Piaget's. In both these studies the results were in

close agreement with Piaget's findings that success developed in three age-related, hierarchically-ordered stages. The threshold of length conservation was found to be seven to eight years of age.

Elkind (1961c) categorized the responses of one hundred children, between the ages of five and eight years, according to Piaget's stages in the development of the ability to include classes. He found that 50 per cent of the age five group were at stage I and 92 per cent of the age eight group were at stage III. Those findings were similar to Piaget's.

Elkind (1964) conducted a study on seriation, numeration and dimensional differences in young children. His findings supported Piaget's thesis that there is a regular increase with age in the child's ability, and the dimensionality of materials affects the ease of success.

A number of studies on length conservation have used the Müller-Lyer Illusion. Murray (1965, 1968a and 1970) presented the results of studies designed to investigate conservation of illusion-distorted lengths. In the 1965 study children were **shown** dotted-lines of equal length, which they saw as being equal. Without being moved the lines were distorted by illusions. The data supported the conclusion that the transition from non-conservation to conservation of length was between seven and eight years of age.

Smedslund (1963b) studied the effects of five procedures on the acquisition of conservation of length by using the Müller-Lyer Illusion. Ninety-six children, ranging in age from five years four months to six years eight months were used in the study. There were two procedures involving practice on addition-subtraction. Another procedure involved a progressive increase in the strength of the Müller-Lyer Illusion, and a fourth procedure involved the anticipation of the outcome of the displacements of the objects. A fifth procedure was a composite of the other four. Smedslund found that the addition-subtraction dominated responses were relatively more frequent with subjects who had achieved conservation, and perception-dominated responses were characteristic of those who did not achieve conservation. He concluded that strengthening the illusion had little effect perhaps because there was a general perception dominance from the beginning of each series of increases in illusion. In another reported study using Müller-Lyer Illusion, Smedslund (1963c) found that conservation of length occurred around eight years of age.

The Müller-Lyer Illusion was also used by Delacy (1967). One hundred and forty primary school children whose ages ranged from six to twelve years were tested. Two 3-inch lines were drawn on white cardboard and labelled A and B. Four sheets of clear plastic were prepared with Müller-Lyer Illusion arrow heads that would fit exactly over

the original lines. Subjects were to view the lines without and with the presence of the arrow heads and were asked the appropriate question regarding the apparent length of the lines. The data from this study indicated that subjects younger than twelve years of age could be classified as non-conservers of length. Delacy noted that this rather advanced age could be partly because of the use of symbolic instead of purely iconic representation (Bruner, 1964).

Similar results were obtained by Glick and Wapner (1968). In this study it is strongly implied that the degree of verbalization affects the age at which children can be classified as conservers of length. They used a verbal and concrete test and found conservation on the verbal test at twelve years and on the concrete test at eight years. However, it ought to be considered that the verbal test used no concrete objects. Perhaps in requiring subjects to justify their answers, the desired performance was not a true meaning of the subject's competency.

The purpose of Coon and Odom's study (1968) was to determine the effects of conformity manipulations on the stability of the transitivity concept in children from seven to fifteen years of age. The results indicated that conformity decreased as age increased. Murray and Youniss (1968) studied the relationship of the achievement of inferential transitivity and serial ordering. They found that only 15 per cent of the children who succeeded on transitivity failed to pass on the

measure of seriation. They concluded: "It follows from our data that seriation is a developmentally prior operation which is an operational forerunner to the understanding of transitivity" (p. 1268). This article came close to highlighting what may be an important ambiguity in measuring and discussing length conservation.

Possible misinterpretation of Piaget's work has been a major point of controversy. Despite articles (Smedslund, 1963c; Murray and Youniss, 1968; Coon and Odom, 1968) on the transitivity of length there appears, in those articles, little awareness of the involvement of any property other than transitivity in the development of the length concept. Smedslund (1963c) outlines a procedure to test concrete transitivity of length in children. However, not until Elkind's study (1967) is there a clear description of two types of conservation inherent in the concept of length conservation. The purpose of Elkind's study was:

... to consider some fundamental aspects of the conservation problem or to demonstrate that every conservation problem assesses two different forms of conservation and that this distinction helps both to clarify Piaget's discussion of conservation and to resolve some of the misunderstandings about conservation that repeatedly occur in the literature on this subject (p. 15).

In this article Elkind distinguishes between identity conservation -- a single comparison -- and equivalence conservation -- a double comparison. He concludes that evidence suggests that identity conservation precedes

equivalence conservation.

Another point which apparently is often forgotten in the literature dealing with conservation is made by Hall and Kingsley (1968). They note that while the child may be unaware of exactly how different characteristics conserve, he may already know that generalization from one kind of conservation to another produces errors. In commenting on Smedslund's experiment involving conservation of weight, Hall and Kingsley indicate the importance of distinguishing between the weight of an object per se and its weight relative to another object. They point out that "if the child thinks the experimenter means a change in the scale is necessary for a change in weight he will learn incorrect responses" (p. 197).

This distinction, similar to Elkind's (1967) definitions of 'identity' and 'equivalence' conservation, is not found in other literature regarding Piaget's work. Consideration of it would probably effect greater similarity in test items on conservation tasks and thereby serve to lessen much of the recurring controversy in this area.

Summary. The studies reviewed above have been attempts to replicate Piaget's investigations by a verbal method. It is evident that various methods produce different results and it is difficult to indicate and even more difficult to obtain a consensus of the ambiguities which may

be inherent in the various methods.

It appears that the threshold age of conservation of length is around seven to eight years of age when verbal methods are employed. The results of Glick and Wapner, and Delacy may be due to the symbolic representations.

Non-verbal

As a deviation from Piaget's method of testing, a number of investigators have used non-verbal methods to measure the acquisition of various concepts.

Braine (1959) is critical of Piaget's variety of stimuli, lack of instructions in length experiments, and language used in testing. In an effort to gain evidence of the undesirable aspects of such a verbal technique as Piaget used, Braine devised a non-verbal method of measuring length conservation. The test used 'uprights' of different lengths which acted as measuring instruments. Children were shown that an upright A was longer than a measuring stick B and that the measuring stick B was longer than an upright C. Children were rewarded, on correct responses, by a candy hidden under the appropriate upright. The test contained some items with only two uprights in order to obtain a measure of their perceptual discriminability. Braine interpreted his data as indicating the threshold age at which 50 per cent of children have transitivity of length is somewhere between four years two months and five years

five months. According to Braine the difference in his results and Piaget's occurred "because in designing his experiments, Piaget fails to eliminate important variables which are not involved in the definition of the processes he sets out to investigate" (p. 16).

Braine (1959) has been severely criticized by Smedslund (1963c). To support his criticism of Braine, he conducted a study using a test which he constructed to test concrete transitivity of length. The data from this study placed the age at which 50 per cent of the subjects possess transitivity of length around eight years. From his data Smedslund confirms, to his satisfaction, the hypothesis that the subjects in Braine's study did not have genuine transitivity. Smedslund's strongest criticism is that Braine failed to eliminate important variables which are not involved in the definition of the process he sets out to investigate. The most serious of these variables Smedslund terms "non-transitive hypotheses" (p. 394). He asserts that theoretical considerations and available data strongly suggest that what Braine observed was not transitivity but non-transitive hypotheses. He suggests that after a number of questions such as involve relative lengths of B and C, the subjects can conclude that the desired answer is always $A > C$ without any indication of genuine transitivity.

Braine (1964) replied to Smedslund's criticism and in turn criticized Smedslund's study of transitivity of length

on the basis that the subjects could not possibly understand his questions. He also presented data as evidence that a non-transitive hypothesis was not used in his experiment. Smedslund (1965) commented on Braine's reply and interprets the new data given by Braine as support for his earlier criticism.

Whether or not the variable of "non-transitive hypotheses" was present in Braine's study is uncertain. Smedslund (1965) concluded:

There appears to exist no compelling evidence for or against the assumption that the performance of Braine's subjects was based only on non-transitive hypotheses (p. 580).

The remarks of Smedslund (1963) -- "The data supports Piaget against Braine" (p. 405), and Braine (1959) "... the results differ from Piaget ..." (p. 16) -- are deserving of comment. Piaget points out that by conservers of length being around seven to eight years of age he means that 75 per cent of the children of that age whom he has studied will conserve length. He continues that around 50 per cent of the children at six years of age will be conservers (1964, p. 31). Both Smedslund and Braine have adhered to 50 per cent rather than 75 per cent in reporting the data in their studies. It may also be inferred from Piaget's writings that he believes children around seven to eight years of age can be classified as conservers of length as he measured this concept and as the concept of length is referred to in adult usage. From

these two observations it is difficult to understand how Smedslund's or Braine's findings, although differing from each other, are in such disagreement with Piaget as they have suggested. Perhaps before the results of any study can be compared with Piaget, the degree to which that study paralleled Piaget's studies ought to be considered.

This sort of comparison or misunderstanding is obvious in a study of conservation of quantity of discontinuous substances undertaken by Silvermann and Schneider (1968). They used an experimental procedure similar to Piaget's but which they labelled a non-verbal method. The subjects were asked to respond to questions by pointing to jars of candy they would take rather than subjects being asked which jars contained the 'most' candy. The child was considered to have conserved if he responded correctly to the question of which of the original jars had more candy and indicated in his explanation that he chose it because it was 'more'. The child was considered not to have conservation if he answered the first question correctly and chose the lesser amount with the same explanation. They found the age at which 50 per cent of the subjects conserved to be around seven years.

The intention was to eliminate the problem of semantics that may have existed in Piaget's experiments. Silvermann and Schneider concluded that their findings conformed very closely to Piaget's and stated that "Piaget's criteria are valid measures of conservation, independent of the child's

capacity for verbal distinctions" (p. 289).

From the information given in this article it appears as if the researcher's method was no less verbal than Piaget's, since the justification given by the subject for his selection was expected to contain the exact words which initially the experimenters intended to eliminate. Despite the apparent intention this can hardly be classified as a non-verbal test of conservation.

Sawada (1966) devised a non-verbal test involving a transformation test of the conservation of length. This test used calipers and Cuisenaire-type rods of multiple and single segmented lengths. The sample consisted of sixty-four children, equal number of both sexes, who ranged in age from sixty-four to ninety-five months. All subjects were given a training session and the response criterion was that the calipers would fit in a certain way as defined by the apparatus used. He found the threshold age of conservation of length to be between five years four months and six years three months. Sawada suggested that if younger children had been included in the study the threshold age level may have been somewhat lower. This study does not suggest what the threshold age of length conservation might have been had a verbal test been used. The data suggest that these results are similar to Braine's(1959). However, since both studies used different apparatus and test items, comparisons or generalizations about them may not be very meaningful.

In an investigation of non-verbal tests of number concepts Wallace (1967) used apparatus resembling that used by Sawada (1966). This consisted of a box-like object with three doors on which were hung three cards containing representations of different numbers. The child was shown a fourth card which represented the same number as one of the cards on the door. The fourth card was not identical to any other card. For example, although it represented the same number, the marks may be of different size than those on the card on the door. The child was asked to find a wooden block by opening the door which indicated the correct response. The non-verbal test was based on Wohlwill (1960). The results supported the view that the attainment of conservation constitutes a landmark in the growth of number concepts. However, Wallace interpreted the results as suggesting that Piaget may have underestimated the part played by counting in the development of conservation. He concluded that the non-verbal test of number concepts used in the study is suitable for the selection of children who have not yet attained conservation of number.

The use of the Müller-Lyer Illusion is reported by Murray (1970) as being used in a verbal and a non-verbal context. The sample in this study consisted of thirty-seven kindergarten and first-grade children. He reports that significantly more conservation responses were found with one verbal procedure and significantly few such responses

with other measures.

It appears that the method and questions used place serious limitations on Murray's study. They are summarized as follows:

(1) In the procedure in the verbal method the subjects were permitted to handle the sticks to compare their relative lengths. This provided the subjects with a certain familiarity with the sticks and a check on their perception. However, no such handling of the materials was permitted in the non-verbal test.

(2) The use of the word 'really' in the question on the verbal test -- for example, "Is this stick really longer than this stick or is it really the same length as this stick or is it really shorter than this stick?" (Murray, 1970, p. 14) -- is suggestive to many children. In their eagerness to please the investigator, if for no other reason, the subjects may be encouraged by the use of such words to answer negatively to the question asked.

(3) The first verbal test involved what Elkind (1967) terms 'identity conservation' -- comparison of two objects, whereas the first non-verbal test involved 'equivalent conservation' -- comparison of three objects. Research has indicated (Elkind, 1967) that identity conservation precedes equivalence conservation.

(4) "A weakness of NV2 (second non-verbal test) is that some 'true' conservers may elect to use the second stick

simply because the opportunity to do so is provided or for some other reason unrelated to the length of the sticks" (Murray, 1970, p. 16).

It might also be noted that a similar study was previously conducted by Murray (1968). This study used very similar apparatus and almost identical questions as Murray (1970). In his discussion of this study Murray (1968) attributes much of the difficulty of the second test (termed non-verbal in the 1970 study) as being,

... consistent with the Genevan observations in the child's development of horizontal decalages; namely, the observation that the same child can exhibit operational stage thinking in one task and yet lag in exhibiting operational thinking and exhibit pre-operational thinking on tasks of more complex content (p. 192).

Summary. Studies using a non-verbal method of measuring conservation have partially avoided the semantic problem; if one did exist in the more verbal methods. However, as is evident in the Braine (1959) and Smedslund (1963) studies, many of the inconsistencies can be attributed to inconsistencies in the criteria that researchers have used to discriminate conservation from non-conservation. Some studies (Braine, 1959; Sawada, 1966) suggest that the threshold age of conservation of length is around five to six years. The results of another study (Murray, 1970) shows fewer conservers on a non-verbal than on a verbal test. Other studies, termed as non-verbal, (Silvermann and Schneider,

1968; Smedslund, 1963; Wallace, 1967) have found that "Piaget's criteria are valid measures of conservation, independent of the child's capacity for verbal distinctions" (Silvermann and Schneider, 1968, p. 289).

II. CONSERVATION AND ACHIEVEMENT

If conservation is to have implications for education it seems appropriate that the relationship between conservation and achievement should be investigated. This section reviews some of the more recent studies in this area.

Dodwell (1960) constructed a test of number concepts which was based on Piaget's work. His sample was a group of thirty-four kindergarten children in Ontario. When the scores on this test were compared, six months later, with the subject's performance on a teacher-made achievement test, the correlation was 0.59. Dodwell concludes that such concept tests would be useful arithmetic readiness for grade one entrants. The value of this study may be limited because of the small group which Dodwell used and the unknown reliability of the teacher-made achievement test.

Hood (1962) devised tests along Piagetian lines. His tests included conservation of number and quantity, seriation, correspondence, and additive composition of classes and numbers. He compared the scores of 126 four to six year old children with teachers' rating on the mathematical ability of the subjects and found a significant

relationship. The reliance on the teachers' ratings, rather than on an objective assessment of subjects' performance, cast some doubt on the findings of this study.

Almy (1966) reports the results of an extensive study including a study of the relationship of conservation and achievement. In the cross-sectional study, including forty-eight second-grade children from a lower class school, Almy concluded that "children who perform well in the conservation tasks also do well in beginning reading and arithmetic" (p. 71). In the longitudinal study the obtained correlation coefficient, ranging from 0.26 to 0.53, was considered by Almy to be sufficiently high to indicate the existence of some relationship between the ability to conserve and mathematics. However, perhaps the findings of this study would have been more meaningful had more variables been controlled.

Steffe (1966) used a conservation of numerosness test in a study involving 341 grade one children. The purpose of the study was to investigate the relationship between the performance on a conservation test and a test constituted of eighteen problems with an additive structure. The effects of intelligence were controlled by placing all the subjects into three categories of intelligence. On the basis of the performance on the conservation test, used as a pretest, children were categorized into levels. The subjects were then administered the problem test. The

analysis indicated that children who had the lowest conservation scores had significantly lower scores on the additive problem test than subjects in the other three levels of conservation.

Reimer (1968) used a conservation test of number, quantity, and length, and the Seeing Through Arithmetic Test as the achievement test. Using a sample of eighty-three grade one students in Edmonton, Alberta, he found a significant correlation ($r=0.41$) between conservation scores and scores on the mathematics achievement test. He concluded that a high achievement test score could be predicted with a high degree of confidence for subjects who had high scores on the conservation test but prediction of mathematics achievement was not possible for subjects who had low scores on the conservation test.

A study which found no significant differences, when I.Q. was controlled for, between conservers and non-conservers in arithmetic achievement was done by Overholt (1964). His sample was made up of fourth grade children. The arithmetic scores were as measured by the IOWA Test of Basic Skills and the conservation tests involved conservation of quantity using plasticine and cotton.

An important consideration arises from this study and may be an inherent factor in other studies of this nature. The achievement tests and the conservation tests may measure completely different aspects of achievement and, therefore,

an insignificant relationship between the two scores might be anticipated. Alternatively, a mathematics achievement test and a test of conservation may load to such an extent on the same factor that a high correlation is inevitable. Since a mathematics achievement test is dependent on the particular curriculum studied by the sample group in a study, perhaps an improved situation would result by including fewer recall items than is often found in achievement tests. The ability to conserve appears to involve more than the ability to recall.

III. SUMMARY

The criteria which discriminate a conserver from a non-conserver are very difficult to define. Elkind (1967) deals with this point in detail. Hall and Kingsley (1968) stress the importance of being cognizant of the various factors inherent in conservation test items. This point of controversy has also led to the Braine and Smedslund interchange. There is evidence (Sawada, 1966; and Glick and Wapner, 1968) that vastly different degrees of verbalizing in questions and responses have established ages of conservation of length from five years four months to twelve years. Thus conservation appears to be more complex than it is regarded by many experimenters. Its importance is not disputed. There are, however, many unresolved questions regarding the criterion necessary to elicit the desired

behaviour.

A purpose of the present study was to answer some questions regarding conservation of length as measured by a verbal and a non-verbal method. Previous comparisons have been made between separate studies and, hence, their value has been limited by this factor. Past research has emphasized a comparison of obtained scores on verbal and non-verbal conservation tasks with little reference to other variables. The relationship between the subjects scores on the verbal and non-verbal methods and their relationships with the variables of age, intelligence, achievement, sex and socio-economic status was investigated in this study. A comparison of the two methods was made on the basis of this investigation.

CHAPTER III

DESIGN OF THE STUDY

I. SAMPLE

The population consisted of the grade one enrollments in four elementary schools in the Edmonton Public School Board System. This comprised a total of about 450 children. The four schools were made available following a request to the Edmonton Public School Board.

The sample of 100 subjects, fifty-two girls and forty-eight boys, was chosen by randomly selecting one classroom from each of the four schools referred to above. By assigning a number to each subject and using a table of random numbers (Hoeh, 1966) the sample was divided into two groups of equal number. One group, twenty-seven girls and twenty-three boys, was administered a verbal conservation test and the other group, twenty-five girls and twenty-five boys, a non-verbal test. Both the verbal and non-verbal tests contained the same number of items and were designed to test the same concepts. The only difference in the two conservation tests was that Cuisenaire-type rods were used in the verbal test whereas in the non-verbal test a response apparatus and Cuisenaire-type rods were used. Both groups were administered the same mathematics achievement test and the Lorge-Thorndike, Level I Form A Intelligence Test.

Table I shows a breakdown of the sample with respect to age and intelligence.

II. INSTRUMENTATION

Mathematics Achievement Test

In this study it was necessary to administer an achievement test at the same time as the conservation tests which were administered during the last week in February and the first three weeks in March. The test which accompanies the Seeing Through Arithmetic (Revised, Hartung et al., 1965) series is intended to be administered at the completion of the appropriate text. Also, this test places more emphasis on recall than was considered desirable for this study. Hence, it was necessary to construct an achievement test for use in the present study.

The achievement test was based on the content of Seeing Through Arithmetic (Revised, Hartung et al., 1965) text for grade one and included test items on topics which subjects in the study had completed by late February.

The first draft of the achievement test was completed after careful examination of the Seeing Through Arithmetic, Book One (Revised, Hartung et al., 1965) and various achievement tests. This draft was administered to a sample of twenty-nine grade one students in late January, 1970. After a study of the item analysis a number of items were deleted from the test and the revised version contained twenty-five

TABLE I
RANGE, MEAN, AND STANDARD DEVIATION OF AGE AND INTELLIGENCE

	Non-verbal Group (n=50)			Verbal Group (n=50)			Total Sample (n=100)		
	Range	Mean	S.D.	Range	Mean	S.D.	Range	Mean	S.D.
Age (months)	74- 94	80.68	4.51	74- 94	79.88	4.32	74- 94	80.28	4.44
I.Q. (Lorge- Thorndike Level I)	73-139	103.72	13.34	74-130	101.96	12.49	73-139	102.84	12.95

items. The reliability as given by the Kuder Richardson Formula 20 was .793. The revised form of the achievement test appears in Appendix A.

The Mathematics Achievement Test consisted of three subtests: (1) Geometry, (2) Numeration, and (3) Addition and Subtraction Facts. The inclusion of these subtests was consistent with the content of Seeing Through Arithmetic, Book One (Revised, Hartung et al., 1965). Table II shows the item numbers that constituted each subtest of the achievement test.

TABLE II
CLASSIFICATION BY SUBTEST OF ITEMS ON THE
MATHEMATICS ACHIEVEMENT TEST

Subtest	Item Numbers	Total Number of Items
Geometry	1, 5, 6, 9, 10, 11, 13, 16, 17	9
Numeration	2, 3, 4, 7, 12, 14, 15	7
Addition and Subtraction	8, 18, 19, 20, 21, 22, 23, 24, 25	9

The achievement test was administered as a group test by the investigator and an assistant. A practice test was given to help clarify the procedure in which the responses were to be made. For each question on the achievement test instructions were read twice. Items 18-25

were done by the subjects after the procedure was briefly explained. No time limit was set on the test. Each subject was given ample time to indicate his responses on the test booklet which he received. In consideration of the age of the subjects in the sample the test was given in two sittings.

Intelligence Test

The Lorge-Thorndike, Level I, Form A Intelligence Test was administered to all subjects in the study. This is a group test using pictorial materials and oral instructions. The test was administered in three separate sittings as recommended by the authors. Care was taken to read instructions slowly and distinctly. An assistant continuously checked to see that subjects were responding to the instructions in correct order. Subjects indicated their responses in the test booklet.

Socio-Economic Status

Blishen's Occupational Class Scale was used as a measure of socio-economic status (Blishen et al., 1968, pp. 741-753). The measure was determined by using the father's occupation, if he was considered to be the head of the household, in reference to the scale which is based on the education and income characteristics of incumbents of these occupations.

The present study used the school records to obtain

the father's occupation. Where this occupation information was missing or vague, as in a small number of cases, no further effort was made to include it. In a few instances the investigator had to make judgements as to similar occupations if the father's occupation was not listed in the Blishen Scale.

Length Conservation Tests

The non-verbal conservation test was constructed by Sawada (1966). This was revised after a pilot study. The test as used in the present study appears in Appendix B.

The verbal conservation test contains the same test items as the non-verbal test but was administered in a verbal context to approximate Piaget's type of interview. The investigator, in deciding on the wording of the questions, was guided by this factor. This test appears in Appendix C.

Both conservation tests consisted of four subtests: (1) Translations, (2) Rotations, (3) Non-Conservation Items, and (4) Illusions. Table III gives a breakdown of the test items constituting each subtest.

Translations. This subtest was designed to measure the child's understanding of the effect of translations on the linear span of an object. Single and multiple segmented lengths were slid a short distance and were either in-plane or out-plane translations as indicated in the tests in Appendices B and C. All items were invariant transformations.

TABLE III
CLASSIFICATION BY SUBTEST OF ITEMS ON LENGTH
CONSERVATION TESTS

Subtest	Item Numbers	Total Number of Items
Translations	1, 2, 3, 4, 5, 6	6
Rotations	7, 8, 9, 10, 11, 12	6
Non-conservation Items	13, 14, 15, 16, 17, 18	6
Illusions	19, 20, 21, 22, 23, 24	6

Rotations. This subtest was designed to measure the child's understanding of the effect of rotations on the linear span of an object. Single and multiple segmented lengths were rotated through various angles. They consisted of in-plane and out-plane rotations. All items were invariant transformations.

Non-Conservation Transformations. The items constituting this subtest consisted of non-invariant transformations. Support for inclusion of this sort of activity can be found in a report of a study by Luchins and Luchins (1964). In four of the items in this subtest .5 cm. of the rod was added or subtracted after the transformation. The plasticine in one item was shortened by approximately .5 cm. by pressing it vertically downwards on

the top of the desk, and in the other item it was lengthened by about .5 cm. by the investigator rolling it under his hand.

Illusions. This subtest was designed to measure the effect of illusions on conservation. The illusions used were variants of the: (1) Müller-Lyer Illusion, (2) Oppel inverted T-Illusion, and (3) the addition of two perspective lines which touched the end-points of one of the rods but not of the other. The illusion cards used in this study are shown in Figure 1.

III. MATERIALS, APPARATUS, AND GENERAL TESTING

PROCEDURE

Materials

The following materials were used in both the verbal and non-verbal length conservation tests:

- (1) Cuisenaire-type rods of one square centimeter in cross-section and of lengths .5, 2, 3, 3.5, 4, 4.5, 9, 9.5 and 10 cm.
- (2) Strips of plasticine of cross-section approximately 5 cm. and 9.5 cm. in length.

Apparatus

The total apparatus consisted of calipers and a response apparatus. This was used only in the non-verbal conservation test, and was the main feature which

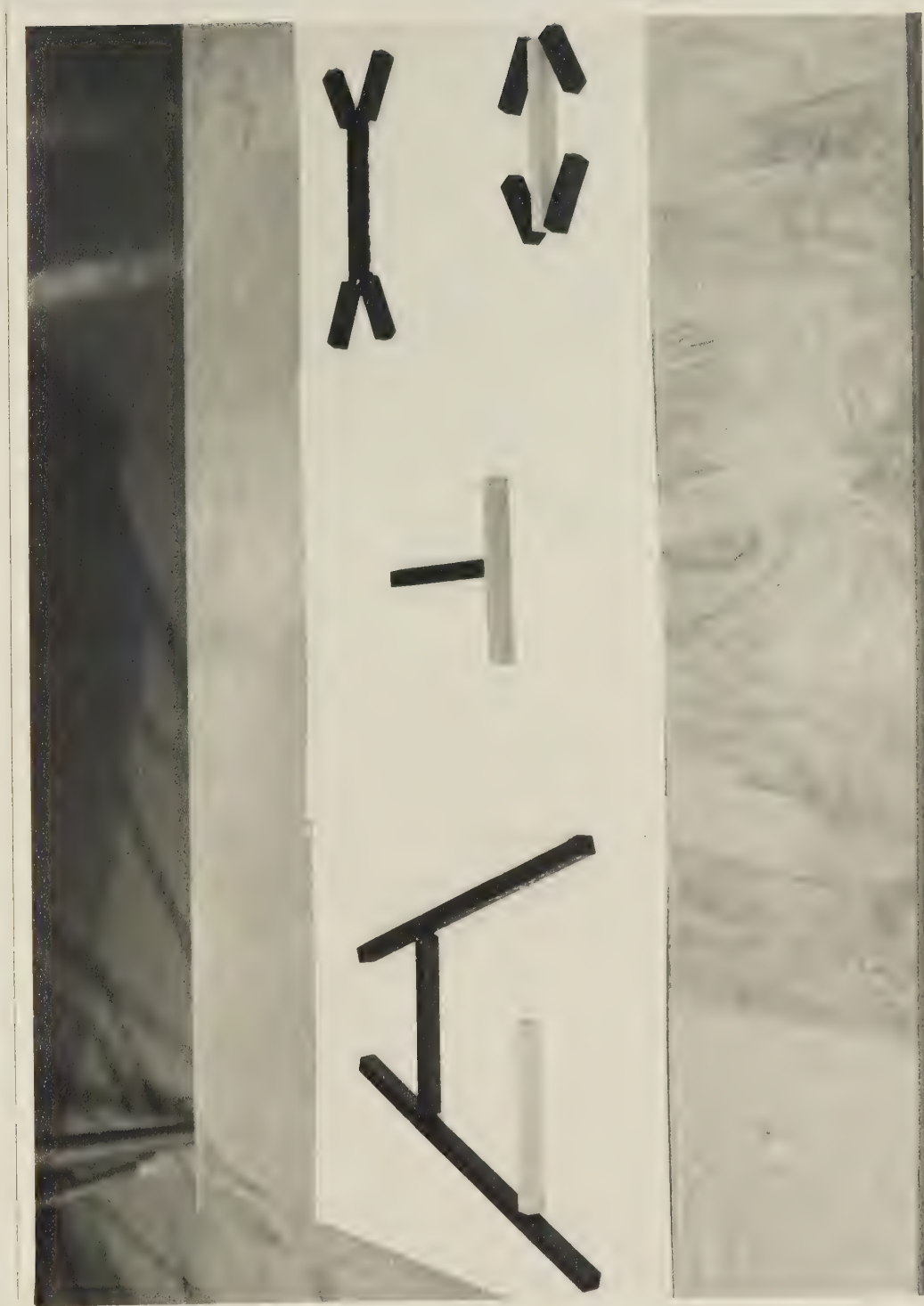


FIGURE 1
ILLUSION CARDS

distinguished the two conservation tests. This apparatus was constructed by Sawada (1966) and was unaltered for use in the present study.

Calipers. Six pairs of calipers were used, having two pairs each to measure lengths 9, 9.5 and 10 cm. respectively. These are shown in Figure 2.

Response Apparatus. The response apparatus consisted of a box with front dimensions 15 cm. by 45 cm. and 15 cm. deep. The box contained three doors through which candies were dispensed as rewards for items answered correctly. On the top of the box, over each door, calipers of different lengths were placed, and a 9.5 cm. rod was inserted in each caliper. This meant that the middle caliper, of span 9.5 cm. contained a rod which exactly fitted it, while the rods in the other calipers were either .5 cm. longer or shorter than the span of the caliper. The apparatus and calipers are illustrated in Figure 2.

Testing Procedure for Non-Verbal Conservation Test

The conservation test was administered individually by the investigator in a private room in the school.

Subjects were first given a training session of approximately five minutes to permit them to become familiar with the response apparatus. Each test item (except the illusion items) was administered as described below.



FIGURE 2

THE RESPONSE APPARATUS: (A) MATERIALS FOR NON-VERBAL
CONSERVATION TEST AND (B) RESPONSE APPARATUS FOR
NON-VERBAL CONSERVATION TEST

A pair of calipers and an object (rod or plasticine as the test item required) were placed in front of the subject. The subject inserted the object in the calipers and was asked to note the 'fit' of the object. The subject was then asked to indicate, by opening the appropriate door on the response apparatus, how the object would fit if it were now applied to the same calipers.

The training session was in sufficient detail that the subject understood the exact criteria he was to use to make decisions. The subject also learned how to use the



FIGURE 3

TESTING ON NON-VERBAL CONSERVATION TEST

apparatus in order to carry out a decision about a particular transformation. First, each subject was shown the response apparatus (Figure 3) and was asked if he had ever played this before. All subjects indicated they were not familiar with this apparatus. The investigator demonstrated the apparatus to the subject by referring to the calipers and rods on the top of the response apparatus and asking if the rods all fit the same. Invariably the subject said no, and when asked to explain the fits, was able to do so. For example, "This one (indicating the shorter one) got a piece gone, this one (indicating the rod equal in length to span of caliper) fits and this one is sticking up over." The investigator then inserted a rod (9 cm.) into a caliper (9 cm. span) and asked the subject to point to the 'fit' on the response apparatus which corresponded to the one the investigator had made. The subject's attention was then directed toward the three doors in the apparatus and the investigator pulled down one door to show the subject how it worked. The subject was then asked to open the other two doors. The subject was told that from then on instead of pointing to the calipers on the apparatus to indicate the corresponding 'fit', the subject should open the door underneath the appropriate caliper. Each correct response would result in finding a candy hidden behind the door.

The subject was then given practice with inserting rods of various lengths in the calipers to enable him to

become familiar with the apparatus. Often the subject volunteered to verbalize about the various 'fits'. The investigator used this terminology in referring to the various situations. After each subject indicated that he had fully understood the procedure to be followed, the next phase of the training session was begun.

This phase required the subject to imagine how the rods would fit the caliper after a transformation. In this phase the subject did the inserting of rods into the calipers. He was told to pay careful attention to what the investigator did and said. "Now I am going to take those sticks that you have fixed here (indicating calipers) and move them like this. (The movement may have been any transformation described in the test in Appendix B.) Now if you put this (calipers) on again in the way you always do, would it be like this, or like this, or like this (pointing successively to the model fits). Find the candy." After two or three questions of this nature the test was begun. No questions in the test were identical to those used in the training session.

The testing procedure for the illusion items (19-24) was as follows.

An orange and a black rod were pushed toward the subject and he was asked, "Are they the same length?" If he appeared not to understand, he was assisted in putting the rods together to determine their relative lengths.

After the subject arrived at the correct aligning procedure and responded to the question regarding their relative lengths, the black rod was removed. The subject was asked to apply the calipers to the orange rod. The calipers always fitted the orange stick in the illusion items. Both sticks were then placed in the illusion cards and the subject was asked, "If you put this (calipers) on this (pointing to the black rod) would it look like this, this, or this (pointing successively at the model fits). Find the candy."

Total testing time lasted about twenty-five minutes.

The 'fits' in the training session and in the test were of three kinds: (1) the object was .5 cm. shorter than the span of the calipers, (2) the object was equal to the span of the calipers, and (3) the object was .5 cm. longer than the span of the calipers.

The calipers were used "to communicate to the child a precise and unambiguous definition of the response criteria" (Sawada, 1966, p. 126).

The child did not measure the same object twice. Thus there was no indication of the conditions under which a re-measurement would give the same results and in this way feedback was easily controlled. Hence, the calipers did not serve to induce the concept of length conservation. It may be argued that the candy reward used in the response apparatus was a means of reinforcement. The reward was used only as an incentive and motivational device and is by no

means unique to the use of calipers or the response apparatus used in this study.

Testing Procedure for Verbal Conservation Test

The materials used in the verbal conservation test -- Cuisenaire-type rods and plasticine -- were identical to those used in the non-verbal conservation test.

The test was administered individually by the investigator in a private room in the school.

A short training session preceded the actual test. Two rods, one orange and one black, of equal length (9 cm.) were slid toward the subject. He was then asked, "Are they the same length?" If the subject had difficulty in correctly aligning the rods, he was assisted by the investigator. After responding correctly as to the relative lengths of the rods, one rod was replaced by rods of 5 cm., 3 cm., and 1 cm. lengths. The subject was asked, "If you put those sticks (indicating 5 cm., 3 cm., and 1 cm. rods) in a straight line, are they the same length as this stick?" (indicating the 9 cm. rod). Again the subject was assisted if he experienced difficulty in correctly aligning the rods. This training continued with various rods until the subject could easily align the rods presented to him (Figure 4). Subjects gained facility in this procedure after two or three examples.

To encourage the subject to think carefully about each response and to help ensure that he was doing his best, he



FIGURE 4

MATERIALS AND SETTING FOR VERBAL TEST

was rewarded with a candy for each correct response. In cases where a subject gave quite a number of incorrect responses he was told to think carefully but that it was important not to feel upset because he was not expected to get them all correct. The investigator felt it important to establish this atmosphere since children at the age of those in the study are generally most anxious to please.

In the first item of the verbal conservation test two rods, one orange and one black, of equal length (10 cm.) were slid toward the subject who was asked, "Are they the same length?" After the subject had correctly aligned the rods -- their end points coterminous -- and had responded regarding their relative length, the subject was asked to pay careful attention to what the investigator did and said. The orange rod was translated a distance of approximately ten inches unto a book. The subject was then asked, "Are they the same length now, or is one longer or shorter than the other?" If there was indication of difficulty in understanding the question, it was repeated once. The order of the components of the question were changed so that each of the three components in the question was placed first an equal number of times.

In the illusion subtest the subject was asked to compare the length of two rods as in the previous test items. Then both rods were placed in the illusion corresponding to the particular test item and the subject was asked, "Are they

the same length, or is one longer than the other?" The wording of the questions varied slightly to prevent the subject from attaching undue importance to the order of the words. Total testing time lasted about twenty minutes.

IV. SCORING PROCEDURES

Achievement Test

This test contained twenty-five items and each item was scored 1 or 0 according to whether the response was correct or not. These raw scores were used in the analysis of the data.

Lorge-Thorndike Intelligence Test

The raw scores were used in conjunction with the subject's chronological age to obtain the intelligence quotient as given in Examiner's Manual, Lorge-Thorndike Intelligence Test.

Verbal and Non-Verbal Conservation Test

As in the previous tests, each item was scored 1 or 0 depending on the response. There was no judgement of degrees of correctness of responses, and any verbalization by the subject was not taken into consideration in determining the score. This study investigated relationships between methods of measuring conservation but did not take the various modes of rationalization into consideration. Subjects who obtained seventeen or more correct responses

were classified as conservers. Those subjects who scored fourteen or less correct responses were classified as non-conservers.

V. CONTROLS BUILT INTO THE TESTS

Achievement Test

The achievement test was administered as a group test by the investigator and an assistant. To aid the subjects to follow items in proper sequence, pictorial symbols were used in the left-hand margin instead of numerals to number the test items. This proved to be more satisfactory than numerals, as used in the pilot study, since subjects did not confuse the number of the test items with a number contained in the item. All the words on the achievement test (Appendix A) were read by the investigator and repeated once. This helped to insure that the subjects clearly heard each word that was spoken in stating the problem to which they were to respond.

Conservation Tests

In an attempt to minimize the error, efforts were made to control forgetting, perceptual cues and guessing. These controls are similar to those suggested by Smedslund (1963c).

In both conservation tests, there was a training session to insure that each subject understood clearly the instructions and, hence, knew the criteria to which he was

expected to respond.

In the non-verbal test, calipers and the response apparatus insured that each subject perceived the initial comparisons. In the training session, each subject was asked to 'tell about' how the sticks fitted the calipers on the response apparatus. Moreover, since the subject always applied the calipers, his active involvement in manipulating the rod into the calipers would increase the probability that he perceived how the rod fit.

In both conservation tests, each time a subject appeared to forget the initial fit or comparison, he was shown it again. The object again underwent the appropriate transformation and the question was repeated.

The following precautions were taken to minimize the effects of guessing:

(1) In the non-verbal conservation test each one of the doors in the response apparatus led to the candy reward an equal number of times.

(2) In both conservation tests there were non-conservation as well as conservation items. This guarded against the possibility of a subject assuming total 'changelessness' situations.

VI. PILOT STUDY

A pilot study, involving ten children, on the conservation tests and involving twenty-nine children on the

achievement test was carried out in late January, 1970. The purposes of this pilot study were to examine the following:

(1) The wording of the verbal questions -- especially the verbal conservation test and the achievement test.

(2) The lengths of the various tests. It was important, for future administering of the tests, to note suitable places where the tests should be divided for short rest periods for the subjects.

(3) The feasibility of broadening the sample to include younger or older children.

On the basis of this pilot study, the final modifications in tests and testing procedures were made.

VII. ANALYSIS

An item analysis was carried out on the mathematics achievement test and both conservation tests. The internal consistencies of the tests were assessed by using the Kuder-Richardson Formula 20.

The relationship between the variables of mathematics achievement, conservation, intelligence, sex, and socio-economic status were found by examining the intercorrelations by means of Multiple Linear Regression Analysis. Where necessary partial correlations were computed and a t-test used to test whether a partial correlation was significantly different from zero.

A t-test was also used to determine which subtests of the conservation tests were superior as predictors of mathematics achievement. A Kolmogorov-Smirnov Test was used to determine if there was a significant difference in the number of subjects who were classified as conservers or non-conservers on the two types of conservation tests used.

CHAPTER IV

RESULTS OF THE INVESTIGATION

This chapter reports an evaluation of the instruments used in the present study. Scatter diagrams and correlation coefficients are shown to indicate the relationship between major variables. The results of testing the hypotheses and other tests made on the data are also reported. The analyses reported in this chapter were carried out on the IBM 360/67 computer using carefully tested programs from the Division of Educational Research Services of the University of Alberta.

I. ACHIEVEMENT TEST

The mathematics achievement test was administered to the total sample used in the study. For this sample the mean of the achievement test was 15.95 and the variance was 13.69. Difficulty coefficients were calculated for each item by dividing the total number of correct responses by the total number of responses given to an item. These are reported in Table IV as percentages.

The items of greatest difficulty, numbers 11, 16 and 17, were all contained in the geometry subtest (Appendix A). Items 2, 3 and 14, whose item difficulty ranged from 83 to 95 per cent, were all contained in the numeration subtest. The addition and subtraction subtest showed greatest

TABLE IV

ITEM DIFFICULTY OF ACHIEVEMENT TEST EXPRESSED AS THE
PER CENT OF SUBJECTS GIVING THE CORRECT RESPONSE
(N=100)

Item Number	Per Cent Successful	Item Number	Per Cent Successful
1	57	14	95
2	87	15	35
3	83	16	20
4	70	17	22
5	71	18	57
6	75	19	72
7	58	20	75
8	60	21	54
9	82	22	77
10	58	23	51
11	31	24	68
12	46	25	48
13	86		

uniformity of item difficulty.

The reliability coefficients for the total test and for each subtest are presented in Table V. A total test reliability of 0.793 was considered to be high enough to accept the scores on the test as indicative of the achievement level of the subjects in the present study.

The distribution of scores on the mathematics achievement test is shown in Figure 5. The distribution was found to be normal by the chi-square goodness of fit

TABLE V
RELIABILITY COEFFICIENTS OF THE MATHEMATICS
ACHIEVEMENT TEST AND ITS SUBTESTS

Test	Number of Items	Reliability Coefficients
Total	25	0.793
Numeration	7	0.426
Geometry	9	0.626
Addition and Subtraction	9	0.820

test. For the total sample the skewness was -0.151 and the kurtosis was -0.364.

The biserial correlations comparing each item to the total test ranged from 0.201 on item five to 0.876 on item nineteen. The mean biserial correlation was 0.466.

II. CONSERVATION TESTS

The verbal and non-verbal conservation tests were each administered to fifty subjects making a total sample of one hundred grade one children. Both conservation tests contained twenty-four items. The tests were made up of the four subtests of translations, rotations, non-conservation items and illusion items (Appendices B and C). The only basic and important difference in the two tests was that one, the non-verbal, was administered using response apparatus

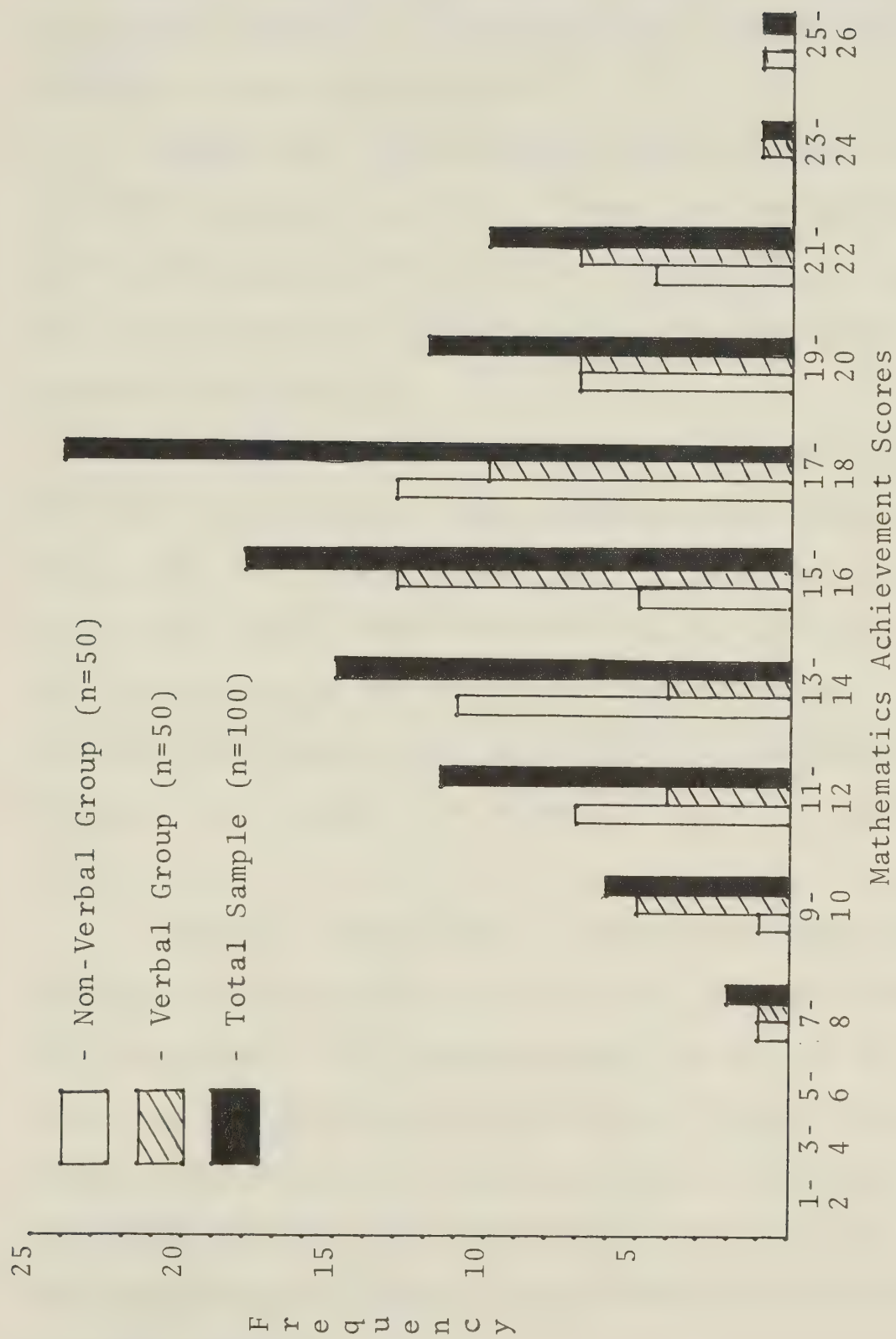


FIGURE 5
DISTRIBUTION OF SCORES ON THE ACHIEVEMENT TEST

and calipers whereas the other, the verbal, did not use any response apparatus other than Cuisenaire-type rods. In the verbal test instructions and the subjects' responses were given verbally. Corresponding items in both tests measured identical concepts.

Difficulty coefficients were calculated for each item on both conservation tests. These difficulty coefficients, giving the proportion of correct responses to each item, and the differences in item difficulty on both tests, are reported in Table VI.

On the verbal test, item one was answered correctly by only 50 per cent of the sample in the verbal group whereas 98 per cent of the non-verbal group gave correct responses to this item. This trend was observed in the early stages of the collection of the data and care was taken to insure that the training session which preceded the testing was adequate. However, the pattern, as shown in Table VI, continued to develop.

Table VI shows that on both tests, item thirteen was answered correctly by only 50 and 57 per cent respectively. This relatively low percentage may be due to the fact that this was the first non-conservation item on the tests. Despite the fact that subjects were asked to pay close attention to what the investigator did and said, apparently many subjects were unable to accommodate the change introduced by this concept.

TABLE VI

ITEM DIFFICULTY OF VERBAL AND NON-VERBAL CONSERVATION
TESTS EXPRESSED AS THE PER CENT OF SUBJECTS GIVING THE
CORRECT RESPONSE

Item Number	Per Cent Successful		Difference
	Verbal (N=50)	Non-Verbal (N=50)	
1	50	98	48
2	82	95	13
3	90	95	5
4	63	91	28
5	74	90	16
6	82	95	13
7	60	97	37
8	90	91	1
9	55	95	40
10	84	97	13
11	84	88	4
12	94	84	-10
13	50	57	7
14	87	59	-28
15	60	67	7
16	71	50	-21
17	40	47	7
18	89	87	- 2
19	40	71	31
20	45	72	27
21	47	84	37
22	72	67	- 5
23	53	69	16
24	68	89	21

Item seventeen presented difficulty to many subjects. The correct responses for both groups were less for this item than for any other item on the entire test. This may be due to the introduction of a new material. In this item plasticine was used instead of Cuisenaire-type rods as used in the previous sixteen items.

There are relatively large differences in the percentage of correct responses on items seven, nine and twenty-one. Perhaps this is pronounced because of a common characteristic of these items. Many subjects in the verbal group seemed to confuse the idea of 'longer' with 'higher' and 'taller'. On examination of items seven, nine and twenty-one it is apparent that this misconception was operative. On the non-verbal test, where the word 'longer' was not used, there was no apparent difficulty. Sawada (1966) also reports negligible difference in difficulty in out-plane and in-plane transformations.

Table VI indicates further that there were for both groups, only slight differences in the difficulty of items involving translations and rotations; that is, items one to six and items seven to twelve respectively.

The non-conservation subtest presented the greatest difficulty for the non-verbal group. Perhaps the element of forgetting was not adequately controlled in this subtest. The verbal group experienced most difficulty with the illusion subtest. It was apparent from the subjects' verbal

responses, that this was due to the contradictory nature of the visual cues. They found it difficult to separate the objects in the illusions from the objects which comprised the illusion form. The question asked the subjects in the non-verbal group -- "How would this (black rod) look if you fit it here (caliper)?" -- may have been sufficient assistance for most of them to visualize the separation of object and illusion form.

The reliability coefficients for the verbal and non-verbal conservation tests and their subtests are presented in Table VII. Reliability coefficients for the conservation tests of .790 and .626 respectively were calculated by using the Kuder-Richardson Formula 20.

TABLE VII

RELIABILITY COEFFICIENTS OF THE VERBAL AND NON-VERBAL
CONSERVATION TESTS AND THEIR SUBTESTS

Test	Number of Items	Reliability Coefficient	
		Verbal	Non-Verbal
Total	24	.790	.626
Translations	6	.513	.546
Rotations	6	.634	.611
Non-conservation Items	6	.407	.495
Illusions	6	.657	.594

It was concluded from the internal consistency as measured by the Kuder-Richardson Formula 20 and from an item analysis of the conservation tests that the tests functioned well using the ability to conserve length as the criterion.

The distribution of scores on both the verbal and non-verbal conservation tests is shown in Figure 6. The means, standard deviations, skewness and kurtosis for both conservation tests are reported in Table VIII.

TABLE VIII

MEAN, STANDARD DEVIATION, SKEWNESS AND KURTOSIS FOR
VERBAL AND NON-VERBAL CONSERVATION TESTS

	Mean	Standard Deviation	Skewness	Kurtosis
Verbal Conservation	15.80	3.87	-0.182	0.010
Non-Verbal Conservation	19.50	2.37	-0.373	0.415

Using a two-tailed t-test it was found that the mean on the verbal conservation test was significantly different from the mean on the non-verbal test at the .01 level of significance.

Subjects who gave fourteen or less correct responses were classified as non-conservers. Those who gave seventeen or more correct responses were classified as conservers.

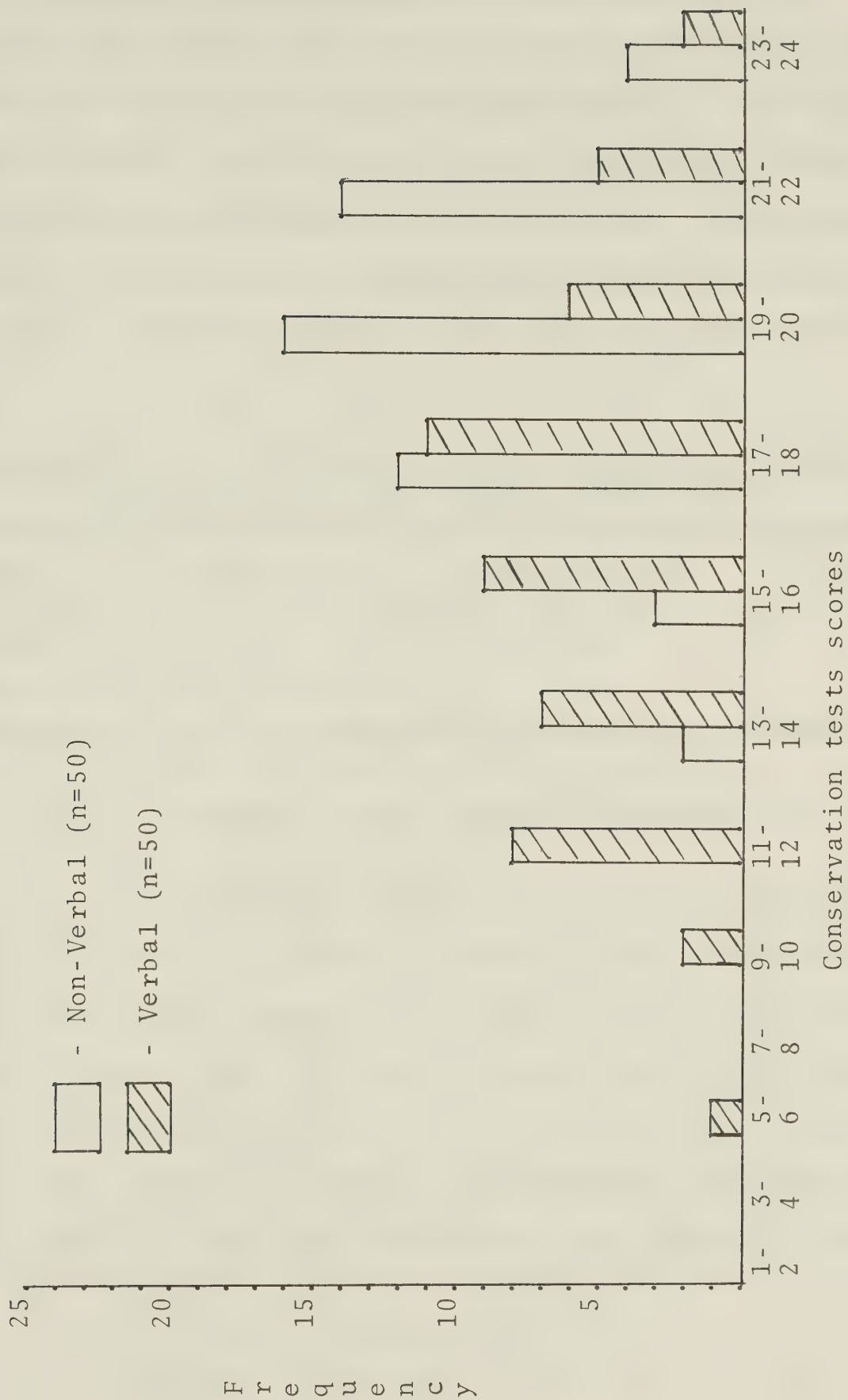


FIGURE 6
DISTRIBUTION OF SCORES ON CONSERVATION TESTS
(N=100)

Table IX presents the number of conservers and non-conservers on both conservation tests. The data in this table was subjected to a Kolmogorov-Smirnov Test and the distributions on the verbal and non-verbal tests were significantly different at the .005 level. (A maximum deviation required for significance at the .005 level was 0.3697. A maximum deviation of 0.3965 was obtained.)

TABLE IX

DISTRIBUTION OF CONSERVERS AND NON-CONSERVERS ON THE
VERBAL AND NON-VERBAL CONSERVATION TESTS

Test	Conserverers	Non-Conserverers	Total
Verbal	23	18	41
Non-Verbal	45	2	47

III. RESULTS OF TESTING THE HYPOTHESES

The correlations among major variables were computed using the DEST 02 computer program. This program calculates the correlations among the variables and the probability ratios associated with these correlations. Correlations and significant probabilities are presented in Tables X and XI. The results of testing the hypotheses are reported in the order in which the hypotheses were stated in Chapter I.

Results of Hypothesis One

There is no significant relationship between the ability to conserve length as measured by:

- (a) a non-verbal conservation test and achievement in mathematics;
- (b) a verbal conservation test and achievement in mathematics;
- (c) a non-verbal conservation test and achievement in mathematics, controlling for the effects of intelligence;
- (d) a verbal conservation test and achievement in mathematics, controlling for the effects of intelligence.

Results. (a) Non-verbal Conservation and Achievement.

Table X indicates that non-verbal conservation correlates significantly with achievement beyond the .05 level of confidence. This relationship is illustrated in the scatter diagram of Figure 7. An examination of the diagram indicates that of the forty-five subjects who were classified as conservers (correct responses ≥ 17) on the non-verbal conservation test, twenty-six (57.8 per cent) obtained an achievement score greater than or equal to the mean of 15.84 of the achievement test. Both subjects who were classified as non-conservers (correct responses ≤ 14) on the non-verbal test failed to obtain a score equal to or greater than the mean

TABLE X
INTERCORRELATIONS AMONG MAJOR VARIABLES OTHER THAN INFORMATION ON VERBAL
CONSERVATION
(Sample = Non-verbal Group)

Variable	1	2	3	4	5	6	7
1. Achievement	1.000						
2. Non-verbal Conservation	0.316*	1.000					
3. Intelligence	0.498**	0.272	1.000				
4. Age	0.102	-0.028	-0.301	1.000			
5. Sex	0.032	0.000	0.017	-0.239	1.000		
6. SES	0.208	-0.085	0.239	-0.212	-0.264	1.000	
7. Conservers on Non-verbal	0.227	0.661**	0.163	-0.245	0.136	0.092	1.000

* Significant at .05 level.

** Significant at .001 level.

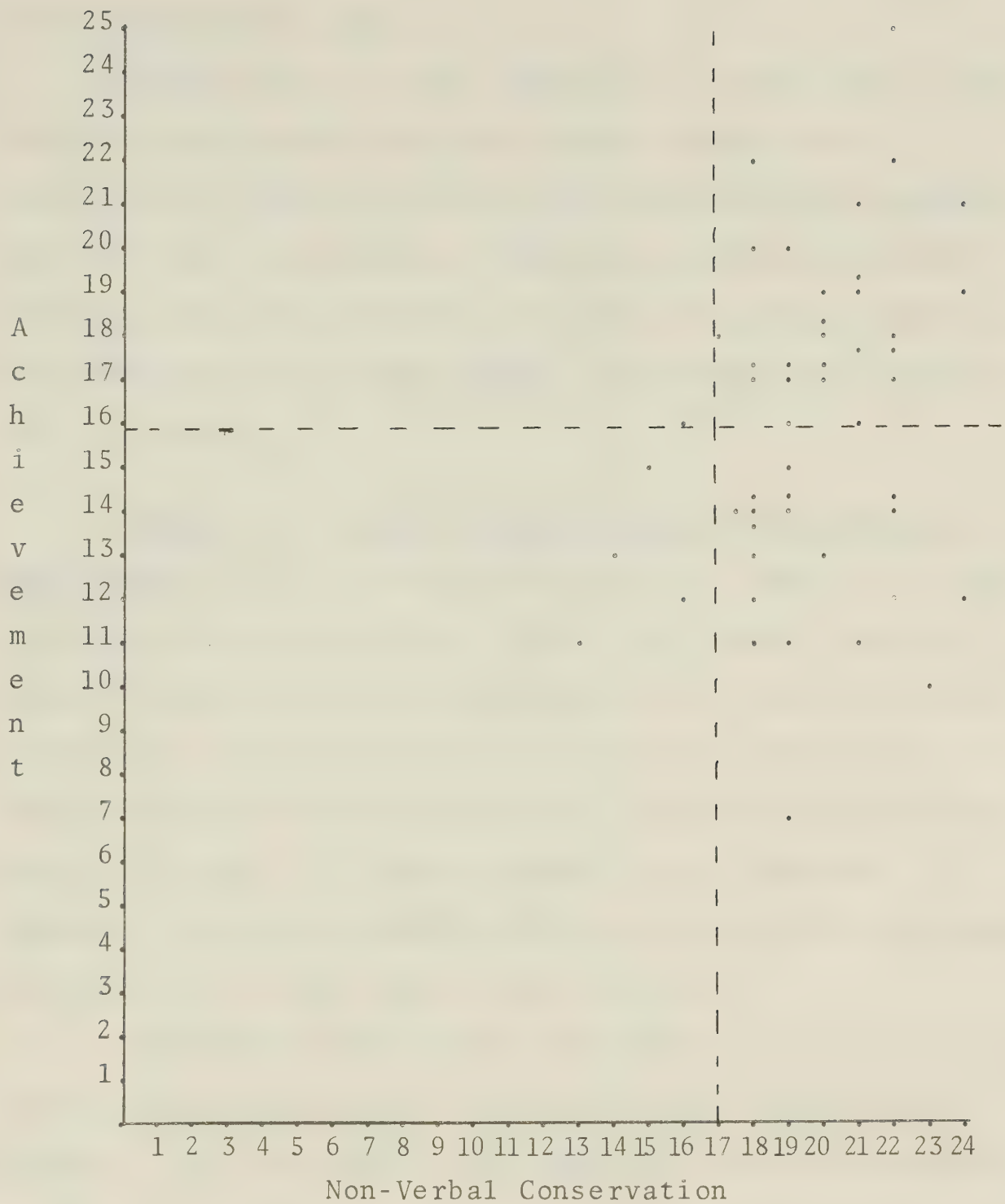


FIGURE 7

SCATTER DIAGRAM FOR NON-VERBAL CONSERVATION TEST SCORES
AND ACHIEVEMENT TEST SCORES

(N=50) (r=0.316)

of the achievement test.

On the basis of these results it is with only a low degree of confidence that achievement scores can be predicted for subjects who had conservation scores greater than or equal to seventeen. Perhaps a similar prediction cannot be made for the non-conservers. The small number of non-conservers on the non-verbal test limits any generalization for this group.

Results. (b) Verbal Conservation and Mathematics Achievement. As is evident from Table XI, there is a significant positive correlation, at the .001 level, between verbal conservation and achievement. This relatively high correlation, compared with the correlation between non-verbal conservation and achievement, may be due in part to the verbal nature of the achievement test. However, it cannot be ascertained whether this verbal factor contributed significantly to the correlations obtained.

The relationship between the verbal conservation test scores and the achievement test scores is illustrated in Figure 8. Of the twenty-three subjects who were classified as conservers (correct responses ≥ 17) on the verbal conservation test, twenty-one (91.3 per cent) achieved a score equal to or greater than the mean (16.06) on the achievement test. Of the eighteen subjects who were classified as non-conservers on the verbal conservation test, fifteen (83.3

TABLE XI
INTERCORRELATIONS AMONG MAJOR VARIABLES OTHER THAN INFORMATION ON NON-VERBAL
CONSERVATION
(Sample = Verbal Group)

Variable	1	2	3	4	5	6	7
1. Achievement	1.000						
2. Verbal Conservation	0.623**	1.000					
3. Intelligence	0.506**	0.555**	1.000				
4. Age	0.129	-0.024	-0.194	1.000			
5. Sex	0.183	0.312*	0.125	-0.203	1.000		
6. SES	-0.043	0.088	0.220	-0.028	0.058	1.000	
7. Conservers on Verbal	0.564**	0.814**	0.530**	0.100	0.325	0.202	1.000

* Significant at .05 level.
** Significant at .001 level.

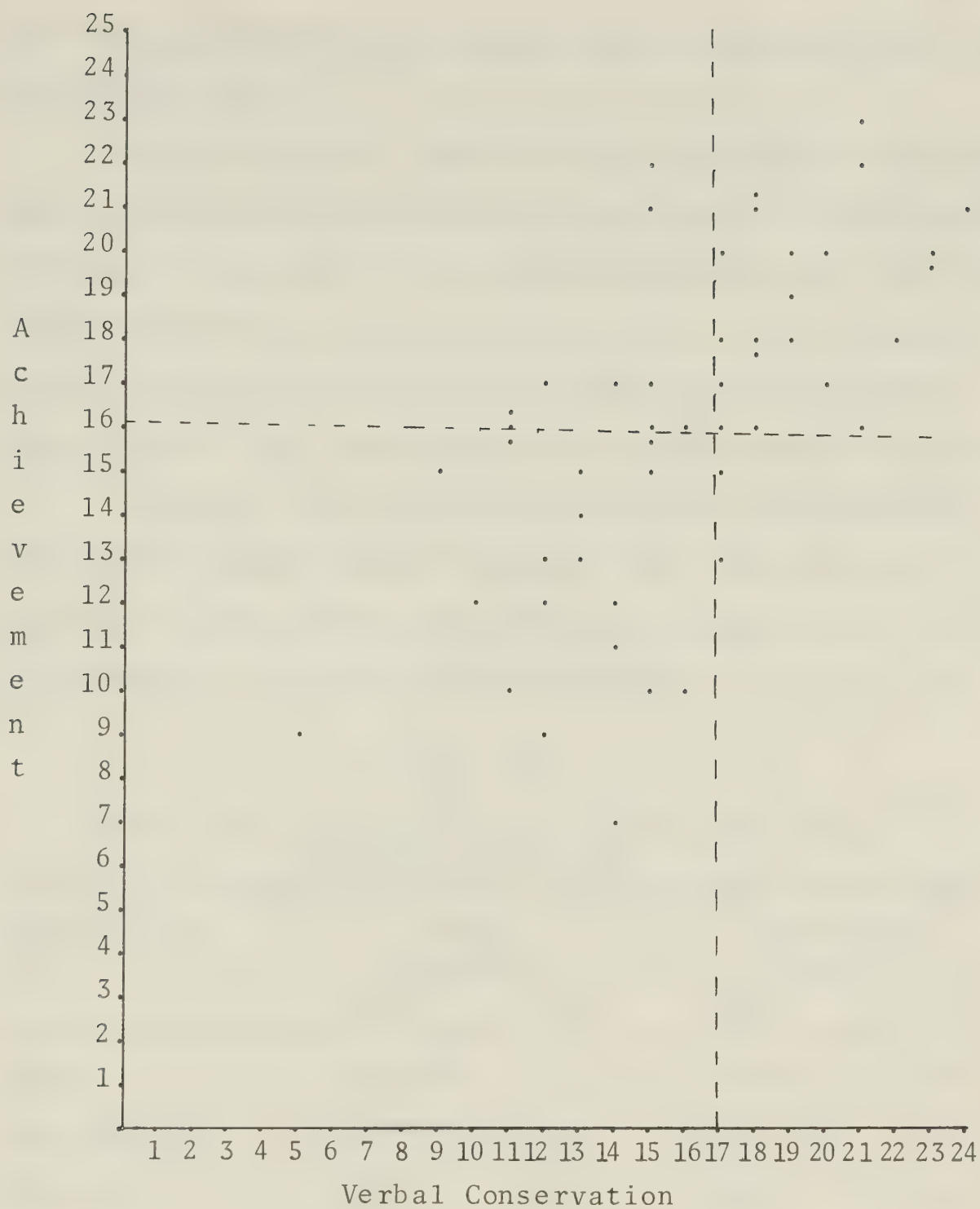


FIGURE 8

SCATTER DIAGRAM FOR VERBAL CONSERVATION TEST SCORES AND
ACHIEVEMENT TEST SCORES

(N=50) ($r=0.623$)

per cent) received a score less than the mean of the achievement test.

On the basis of these results it is with a very high degree of confidence that achievement scores can be predicted for subjects who were classified as conservers on the verbal conservation test. Achievement scores can also be predicted with a high degree of confidence for subjects who were classified as non-conservers on the verbal conservation test.

Table XII indicates that the verbal conservation subtests of translations, rotations and illusions may be superior to the subtest containing non-conservation items as predictors of mathematics achievement.

TABLE XII
CORRELATIONS OF CONSERVATION SCORES WITH TOTAL
ACHIEVEMENT SCORES

Conservation Tests	Verbal (N ₁ =50)		Non-Verbal (N ₂ =50)	
	Corr.	<u>t</u> ratio	Corr.	<u>t</u> ratio
Total	.6225***	5.511	.3156*	2.304
Translations	.3703**	2.762	.2441	1.744
Rotations	.4061**	3.079	.2708	1.949
Non-conservation Items	.3286*	2.410	.2666	1.916
Illusions	.3839**	2.880	.2394	1.708

* Significant at the .05 level ($t \geq 2.011$)

** Significant at the .01 level ($t \geq 2.681$)

*** Significant at the .001 level ($t \geq 3.506$)

Results. (c) and (d) Conservation and Achievement
Controlling for the effects of I.Q. The correlation coefficients referred to in the preceding pages of this chapter have indicated the relationship between two variables. However, it is unlikely that a realistic situation exists in which only two variables are operative. In this section partial correlations involving the major variables of achievement, conservation, and intelligence are discussed.

It was earlier indicated (Tables X and XI) that a significant correlation existed between conservation, both verbal and non-verbal, and achievement. The partial correlation coefficients between conservation scores and achievement scores when the differential effects of I.Q. are controlled for are given in Table XIII. This table indicates that knowledge of conservation scores on a verbal conservation test significantly improves the ability to predict achievement scores made on the basis of I.Q. scores. Knowledge of the conservation scores on a non-verbal conservation test does not permit a significantly improved prediction of mathematics achievement scores made on the basis of I.Q. scores.

TABLE XIII

PARTIAL CORRELATIONS OF CONSERVATION SCORES WITH
ACHIEVEMENT SCORES CONTROLLING FOR EFFECTS
OF I.Q.

Achievement	Conservation	Partial Correlation	<u>t</u> -ratio
Total	Verbal	.476*	3.750
Total	Non-verbal	.216	1.533

*Significant at the .001 level ($t \geq 3.506$).

Conclusion. On the basis of these results, parts (a), (b) and (c) were rejected. There was a significant relationship between conservation, both verbal and non-verbal, and achievement in mathematics. There was also a significant correlation between non-verbal conservation and achievement when the effects of intelligence were controlled.

Part (d) of Hypothesis One was accepted. When the differential effects of I.Q. were controlled for there was not a significant correlation between non-verbal conservation and mathematics achievement.

Results of Hypothesis Two

There is no significant relationship between:

- (a) mathematics achievement and sex;
- (b) ability to conserve length as measured by a non-verbal test and sex;
- (c) ability to conserve length as measured by a verbal test and sex;
- (d) mathematics achievement and socio-economic status.

Results. (a), (b) and (d). There was no significant relationship between sex and mathematics achievement or length conservation as measured by a non-verbal test. The correlation between mathematics achievement and socio-economic status was not significant. The data on which this information is based is reported in Table XIV.

Results. (c). Table XIV indicates a significant positive correlation, at the .05 level, between sex and conservation as measured by a verbal test.

Conclusion. Parts (a), (b) and (d) of Hypothesis Two were accepted. Sex did not correlate significantly with mathematics achievement and non-verbal conservation. There was no significant correlation between mathematics achievement and socio-economic status.

Part (c) of this Hypothesis was rejected. There was a significant correlation between sex and verbal conservation.

TABLE XIV

CORRELATIONS OF ACHIEVEMENT AND CONSERVATION TESTS
WITH SEX AND S.E.S.

Tests	Sex	S.E.S.
Mathematics Achievement	.086	.103
Non-verbal Conservation	.0	-.085
Verbal Conservation	.312*	.088

* Significant at the .05 level.

Results of Hypothesis Three

There is no significant relationship between:

- (a) the ability to conserve length as measured by a non-verbal conservation test and intelligence;
- (b) the ability to conserve length as measured by a verbal conservation test and intelligence;
- (c) achievement in mathematics and intelligence, controlling for the effects of non-verbal conservation scores;
- (d) achievement in mathematics and intelligence, controlling for the effects of verbal conservation scores.

Results. The data in Table X (page 68) indicates no significant correlation between non-verbal conservation and intelligence. Table XI (page 71) indicates a significant correlation at the .001 level between verbal conservation and intelligence.

In order to ascertain whether I.Q. is significantly correlated with mathematics achievement when the differential effects of conservation are controlled for, partial correlations between achievement scores and I.Q. were calculated. These are reported in Table XV. This table indicates that the knowledge of I.Q. scores does not contribute significantly to the prediction of achievement scores when the effects of verbal conservation scores are

controlled for. However, the partial correlation was significant at the .001 level between achievement and intelligence when the effects of non-verbal conservation were controlled for.

TABLE XV

PARTIAL CORRELATIONS OF ACHIEVEMENT SCORES WITH I.Q.
CONTROLLING FOR THE EFFECTS OF CONSERVATION SCORES

Achievement	I.Q.	Partial Correlation	<u>t</u> -ratio
Total	Verbal Group	.246	1.758
Total	Non-verbal Group	.452*	3.511

* Significant at the .001 level ($t \geq 3.506$).

Conclusion. Part (a) of this hypothesis was accepted. There was no significant correlation between non-verbal conservation and intelligence.

Parts (b) and (c) were rejected. There was a significant correlation at the .001 level between intelligence and verbal conservation and between intelligence and achievement when the effect of non-verbal conservation scores was controlled for.

Part (d) was accepted. There was no significant correlation between achievement scores and intelligence when the effect of verbal conservation scores was controlled for.

The results of this hypothesis seem to imply that

some verbal component, either in an I.Q. test or in a verbal conservation test, is necessary to obtain a high degree of confidence in predicting achievement scores from I.Q. or conservation scores. It further indicates that achievement scores can be predicted with a high degree of confidence if verbal conservation scores are known and effects of I.Q. scores are controlled for.

Results of Hypothesis Four

There is no significant difference between the mean score on the non-verbal length conservation test and verbal length conservation test.

Results. The mean scores on the non-verbal length conservation test and verbal length conservation test were significantly different at the .005 level. This information is reported in Table XVI.

Conclusion. This hypothesis was rejected. Although, the subjects who were administered the verbal and non-verbal conservation tests formed two distinct groups, the information given in Table I of Chapter III indicates the groups were not significantly different with respect to age, or intelligence.

TABLE XVI
COMPARISON OF TWO GROUPS ON ACHIEVEMENT AND CONSERVATION TESTS

Test	Non-verbal Group		Verbal Group		Total Group	
	Range	Mean	S.D.	Range	Mean	S.D.
Achievement	7-25	15.84	3.59	7-23	16.06	3.81
Conservation	13-24	19.50*	2.37	5-24	15.80*	3.87

* Significantly different at the .005 level.

IV. SUMMARY

This chapter contains an analysis of the various instruments used in the study, and the results of testing four hypotheses which were associated with the major purposes of the study as outlined in Chapter I.

The first purpose of the study was to measure the concept of length conservation by using a verbal and a non-verbal method. It was found that subjects who were administered the non-verbal conservation test obtained a significantly higher conservation score than the subjects who were administered the verbal conservation test.

The major purpose of the present study was to compare the verbal and non-verbal methods of measuring length conservation as predictors of mathematics achievement. Both the verbal and non-verbal methods were significantly related to achievement. However, the results indicated that mathematics achievement scores can be predicted with a higher degree of confidence when the verbal conservation scores are known as opposed to knowledge of non-verbal conservation scores. If the effects of I.Q. are controlled for there does not remain a significant correlation between mathematics achievement and non-verbal conservation but there remains a significant correlation, at the .001 level between verbal conservation and mathematics achievement.

Implications arising from these findings will be discussed in the next chapter.

CHAPTER V

SUMMARY, DISCUSSION, IMPLICATIONS, AND RECOMMENDATIONS

I. SUMMARY OF THE INVESTIGATION

The present study was designed primarily to investigate the role of conservation as a predictor of mathematics achievement. Attention was focused on verbal and non-verbal techniques of gauging length conservation (Appendices B and C). These gauging techniques were studied for two reasons: (1) to establish, for the purpose of this study, which technique -- verbal or non-verbal -- obtained a higher mean score, and (2) to investigate the relationship of the scores resulting from each technique to the important variables of intelligence and mathematics achievement.

In order to gather the necessary data a sample of grade one children was selected and several instruments were employed.

Sample

The sample of one hundred grade one children, forty-eight boys and fifty-two girls, was chosen by randomly selecting one grade one classroom from each of four schools within the Edmonton Public School system. The ages of these children ranged from seventy-four to ninety-four months. The intelligence quotients, as measured by the Lorge-Thorndike

Intelligence Test, Form A, Level I, ranged from 73-139.

It was assumed that these children were representative of urban children around six to six and one-half years.

Instruments

A mathematics achievement test (Appendix A) was constructed by the investigator for use in the present study. This test consisted of twenty-five items constituting the three subtests of Geometry, Numeration, and Addition and Subtraction Facts. This was administered to the complete sample as a group test. The KR-20 reliability was 0.793.

Both conservation tests (Appendices B and C) were administered individually. One half of the sample, twenty-five boys and twenty-five girls, were administered the non-verbal test and the other half of the sample, twenty-three boys and twenty-seven girls, were administered the verbal test. Both tests were designed to measure the concept of length conservation in young children. The distinguishing feature of the tests was the context in which they were administered. In the non-verbal test calipers and response apparatus were used in addition to the Cuisenaire-like rods used in the verbal test. The KR-20 reliability for the non-verbal test was 0.626 and for the verbal test the KR-20 reliability was 0.790.

Most of the analysis of the data was done by using computer programs supplied by the Division of Educational

Research Services, Faculty of Education, at the University of Alberta.

Conclusions

A summary of the findings will be presented on the basis of testing the hypotheses.

It was found that a significant correlation existed between the ability to conserve length as measured by a non-verbal conservation test and achievement in mathematics. A significant correlation also existed between the ability to conserve length as measured by a verbal conservation test and achievement in mathematics. When the effects of intelligence were controlled for there remained a significant correlation between the ability to conserve length as measured by a verbal conservation test and mathematics achievement. However, there was no significant correlation between non-verbal conservation and mathematics achievement when the differential effects of intelligence were controlled for.

On the basis of the above findings parts (a), (b) and (d) of Hypothesis One were rejected. Part (c) was accepted.

No significant relationship was found between mathematics achievement and sex or between the ability to conserve length, as measured by a non-verbal test, and sex. It was found, however, that a significant correlation existed between the ability to conserve length as measured by a

verbal test and sex. The relationship between mathematics achievement and socio-economic status was not significant.

Parts (a), (b) and (d) of Hypothesis Two were accepted. Part (c) was rejected.

The ability to conserve length as measured by a non-verbal test did not correlate significantly with intelligence. There was a significant correlation at the .001 level between verbal conservation and intelligence. It was found that the intelligence scores did not contribute significantly to the prediction of achievement scores when the effects of verbal conservation scores were controlled for. However, there was a significant correlation at the .001 level between mathematics achievement and intelligence, controlling for the effects of verbal conservation scores.

Parts (a) and (d) of Hypothesis Three were accepted. Parts (b) and (c) were rejected.

The mean scores on the non-verbal length conservation test, mean of 19.50, and the verbal length conservation test, mean of 15.80, were significantly different at the .005 level. Hypothesis Four was, therefore, accepted.

II. DISCUSSION AND IMPLICATIONS OF THE FINDINGS

The findings indicated that mathematics achievement scores can be predicted with a high degree of confidence by using verbal conservation of length scores independent of the effects of intelligence scores. A significant correlation

was found between mathematics achievement and verbal conservation when the effects of age, intelligence, sex and socio-economic status were included. However, when the effects of intelligence were controlled for, by a partial correlation procedure, there remained a significant correlation between mathematics achievement and verbal conservation. Furthermore, a significant correlation was found between mathematics achievement and intelligence before but not after controlling for the effects of verbal conservation.

It was found that mathematics achievement scores cannot be predicted with as high a degree of confidence by using non-verbal conservation scores as by using verbal conservation scores. Mathematics achievement correlated significantly with non-verbal conservation. However, this significant correlation did not remain when the effects of intelligence were controlled for.

It is, therefore, evident that verbal conservation scores were found to be superior to the variables of intelligence and non-verbal conservation test scores as a predictor of mathematics achievement at the grade one level. Intelligence test scores and non-conservation scores were also found to be significant but were inferior to verbal conservation scores as predictors of mathematics achievement.

The importance of verbal conservation scores as a predictor of mathematics achievement is the focal point of

implications arising from this study. It is important to remember that in the present study the conservation test involved only the length concept. The findings showed that mathematics achievement can be predicted with a high degree of confidence on the basis of conservation of length. Perhaps by using a verbal conservation test involving a number of conservation tasks an even more accurate prediction of mathematics achievement can be made.

There was nothing in the findings of the present study to indicate that the conservation tests used for the purpose of predicting mathematics achievement should be long or exhaustive. The low correlation between the subtest of non-conservation items and mathematics achievement may suggest that this subtest be eliminated from future conservation tests. Perhaps two or three test items on each concept being measured may prove sufficient. However, there was no exhaustive effort in the analysis of the data to find support for this conjecture.

The mean score on the non-verbal conservation test was significantly higher than that on the verbal conservation test. This finding appears to be consistent with that of Braine (1959) and Sawada (1966). If the criterion for a conserver is the obtaining of a certain score on a conservation test, then the non-verbal test will indicate a much greater number of conservers than does the verbal conservation test. This does not, however, support an argument for the use

of a non-verbal conservation test as a predictor of mathematics achievement. It was earlier reported that there was no significant correlation between non-verbal conservation and mathematics achievement when the effects of intelligence were controlled for.

This does not suggest that a non-verbal conservation test can serve little or no purpose. A non-verbal technique has an important place in gauging conservation. It may be used as a diagnostic device to detect the presence of concepts in young children at an earlier age than the verbal technique of measuring length conservation. The subjects' responses on such a non-verbal test may indicate some beneficial activities in which a child might engage to aid in the transition from a conserver on a non-verbal test to a conserver on a verbal test. Varied experiences with concrete materials might well be one such area to be considered in providing activities to aid in this transition.

The findings of this study indicated that if the child were classified as a conserver on a non-verbal conservation test it did not mean that he could verbalize about the concept which was being measured. On the other hand, many of the subjects who were classified as non-conservers on the verbal conservation test because of their inability to verbalize may have been classified as conservers on a non-verbal conservation test. If instruction to grade one children and the method of measuring mathematics

achievement remains to a very large degree in a verbal context it is unlikely that a non-verbal conservation test will serve as a good predictor of mathematics achievement.

A verbal factor seemed to permeate the findings of the present study. The non-verbal techniques enabled more children to express their ability to conserve than did the verbal technique. It was noted that a semantic problem may have been present in terms on the verbal conservation test (Appendix C) involving out-plane transformations. It was apparent from the subjects' responses that they equated words like 'higher', 'taller', and 'larger' to the word 'longer'. Children at the grade one level often show difficulty in understanding questions and instructions from adults. Their verbalization, often not an accurate description of their thoughts, may indicate a lack of knowledge or misunderstanding.

The prominence of a verbal factor is not surprising. At present most instruction which a grade one pupil receives is presented in a verbal context. When a check is made to see if the child has benefited from that instruction it is often an exercise in verbalization. Failure to verbalize is often equated with failure to learn. This apparent importance of a verbal factor suggests that a deliberate effort should be made to provide experiences with manipulative materials which emphasize distinctions between such words as 'longer', 'taller', 'higher', 'farther', and 'nearer'. If a

firm basis is to be established for mathematics this emphasis on precise terminology is important.

The findings of a significant correlation between sex and scores on the verbal conservation test is consistent with those of many previous studies.

McCarthy (1959) writes:

One of the most consistent findings to emerge from the mass of data accumulated on language development in American white children seems to be a slight difference in favour of girls in nearly all aspects of language that have been studied (p. 77).

Karlin (1947) attributes the sex difference in speech development to the later psychological and physiological development in boys.

There was no significant correlation found between sex and mathematics achievement scores and scores on the intelligence test. It may be that the verbal factor in the verbal conservation test was operating to a greater extent than in the other tests mentioned.

No effort was made to investigate the relationship between age and the other variables. Since subjects in the sample were in grade one with an age range of seventy-four to ninety-four months, it is unlikely that any significant correlations would have been revealed. There was no significant relationship between mathematics achievement and socio-economic status. Perhaps this was in part due to the fairly homogeneous group which comprised the sample. Other

studies (Reimer, 1968; Cathcart, 1969) report similar findings.

III. RECOMMENDATIONS FOR FURTHER RESEARCH

This study used only a length conservation test and a sample of grade one children. One recommendation for further research is a study using various conservation tasks in both a verbal and a non-verbal context. The tests, both in the verbal and non-verbal form, might be alternately administered to the complete sample of the study. This would provide a more detailed comparison of both measuring techniques of conservation concepts.

A second recommendation for further research is the development of a non-verbal instrument to measure mathematics achievement. This non-verbal instrument might then be used to investigate the relationship between mathematics achievement and non-verbal conservation.

The verbalization aspect seems to be prominent in many studies involving conservation. A third recommendation for further research is an investigation into language comprehension in relation to mathematics achievement. This may help to provide information on the meaning which children attach to verbalization in mathematics at the grade one level.

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A P P E N D I X A

MATHEMATICS ACHIEVEMENT TEST

MATHEMATICS ACHIEVEMENT TEST

Name _____

Put an 'X' beside the picture that shows the turtle has only one way of getting out of the corral.



Put an 'X' in the box by the numeral which is one greater than 19.


☐ 20

☐ 18

☐ 100


Put an 'X' in the box which shows how much money is in the picture.

☐ 5

☐ 10

☐ 1

☐ 105¢

☐ 11¢

☐ 16¢


Put an 'X' in the box to show the numeral that the picture makes you think of.

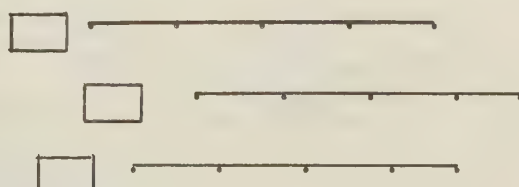
☐ 8 hundreds 7 tens

☒ 87¢

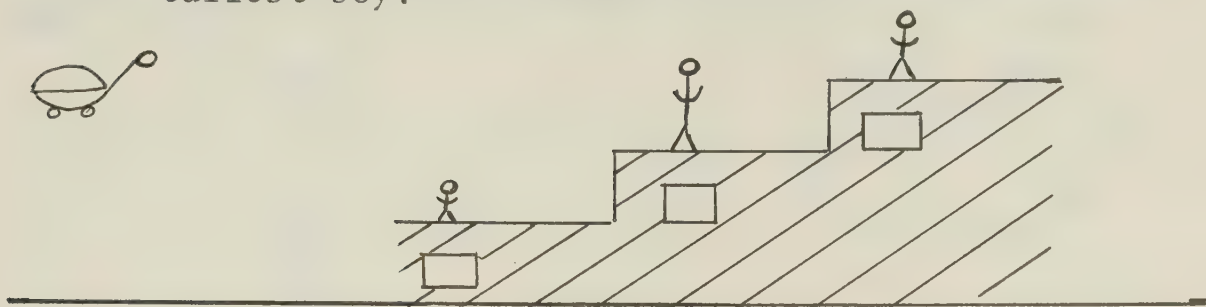
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☐ 8 tens 7 ones

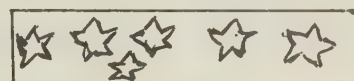
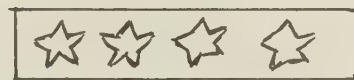
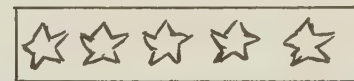

Put an 'X' in the box beside the longest line.



Put an 'X' beside the picture that shows the tallest boy.



Find the box that has the same number of stars as this.


☐

☐

☐


Bobby's pet puppy weighed just 3 pounds at first. Now the puppy weighs twice as much. How much does the puppy now weigh?

☐ 6

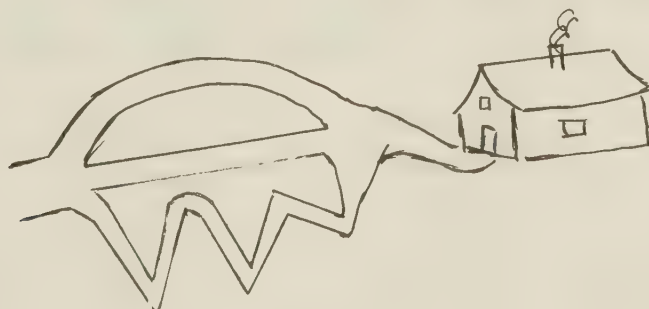
☐ 9

☐ 5


All three bottles are filled with candy. Put an 'X' on the container which has the most candy.



Put an 'X' on the longest road to Bobby's house.

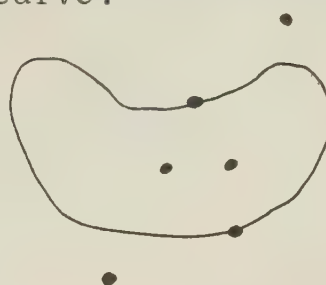




Put an 'X' beside the box which tells how many points are shown inside the curve.

☐ 2

☐ 4

☐ 6


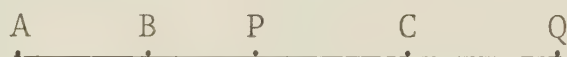
Put an 'X' beside the numeral that has the smallest value.

☐ 19

☐ 15

☐ 13


Put an 'X' in the box that names two points that P is between.


☐ A and B

☐ B and C

☐ C and Q


Decide which number is shown by the tally marks and put an 'X' in the box.

☐ 56

☐ 65

☐ 11

tens	ones
111	1111
11	11



3 is between a number and 9. Put an 'X' in the box by this number.

☐ 4

☐ 6

☐ 2

Put an 'X' in the box by the numeral that tells how many sides a triangle has.

☐

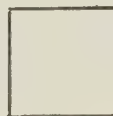
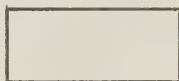
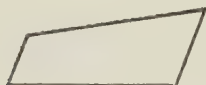
2

☐

3

☐5

Put an 'X' on the square.



FOR EACH OF THE OTHER EXERCISES, FIRST DECIDE WHICH NUMERAL BELONGS WHERE THE SCREEN IS. THEN MARK YOUR ANSWER BY PUTTING AN 'X' IN THE BOX BESIDE THE NUMERAL.

1		6	
7	$5 - 4 = \begin{array}{c} _ \\ _ \\ _ \end{array}$	7	$5 + 2 = \begin{array}{c} _ \\ _ \\ _ \end{array}$
9		8	
4		8	
5	$4 + 4 = \begin{array}{c} _ \\ _ \\ _ \end{array}$	7	$8 - 7 = \begin{array}{c} _ \\ _ \\ _ \end{array}$
8		1	
8		2	
6	$7 + 1 = \begin{array}{c} _ \\ _ \\ _ \end{array}$	0	$2 + 0 = \begin{array}{c} _ \\ _ \\ _ \end{array}$
5		3	
5		2	
4	$9 - 4 = \begin{array}{c} _ \\ _ \\ _ \end{array}$	8	$9 - 7 = \begin{array}{c} _ \\ _ \\ _ \end{array}$
8		3	

A P P E N D I X B

NON-VERBAL CONSERVATION TEST

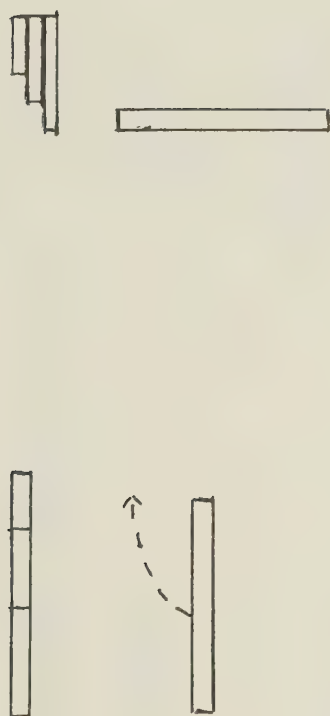
A NON-VERBAL TRANSFORMATION TEST OF THE CONSERVATION OF LENGTH

Test Item	Materials	Original Configuration	Transformed Configuration	Transformation Remarks
SUBTEST I. TRANSLATIONS				
1.	10C* 10R**			Out-plane translation
2.	9C (3, 3, 3)R			Translate as indicated (in-plane)
3.	9C (3, 3, 3)R			Translate as indicated (out-plane)
4.	9.5C 9R			Translate as indicated (out-plane)
5.	9.5C 9R			Out-plane translation

* C = Calipers
** R = Rods

Translate as
indicated
(out-plane)

90° out-plane
rotation



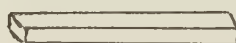
90° out-plane
rotation

Rotate as
indicated
(in-plane)



SUBTEST II. ROTATIONS

8. 9.5C
(3,3,3)R



9. 9.5C
9R

10. 9.5C
10R

11. 9.5C
(5,3,2)R

12. 9.5C
(5,3,2)R

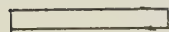


7. 10C
10R



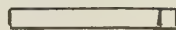
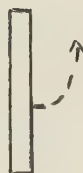
SUBTEST III. NON-CONSERVATION TRANSFORMATIONS

13. 9.5C
(.5,9)R



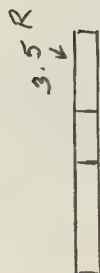
90° in-plane rotation followed by subtraction of .5 cm. rod

14. 9.5C
(.5,9.5)R



90° in-plane rotation followed by addition of .5 cm. rod

15. 10C
(5,2,3,3.5)R
3R is black
3.5 is white



Align 3 and 3.5 rods in identity position. Then replace 3 rod with 3.5 rod

16. 10C
(5,3,2)R
(4.5,3,2)R
5 is black
4.5 is white



Align (5,3,2) and (4.5,3,2) in identity position. Then remove (5,3,2).

17. 9.5C
9.5R
Plasticine



Pound one end of the plasticine in about .5 cm.

18. 9.5C
9.5R



Roll the plasticine under the palm so that it gets slightly longer.

SUBTEST IV. ILLUSIONS

19.

10C
10 B*
100r**



Place the rods
into the
illusions forms
by any combin-
ations of trans-
lations and
rotations.

20.

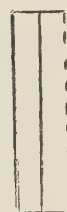
10C
10B
100r



As for item 19

21.

10C
10B
100r



As for item 19

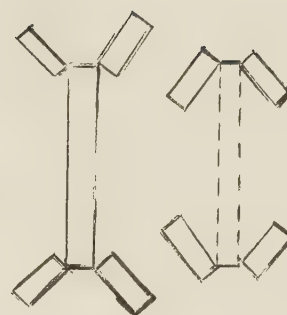
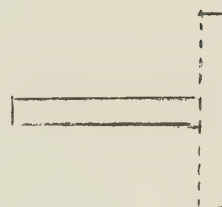
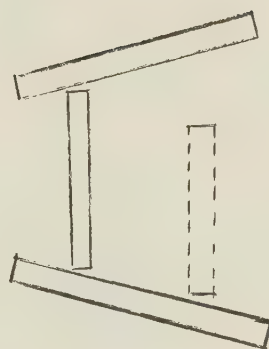
22.

10C
9.5B
10 Or



As for item 19

* B = Black
** Or = Orange

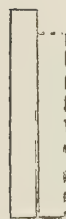


23. 9.5C
10B*
9.5 Or**

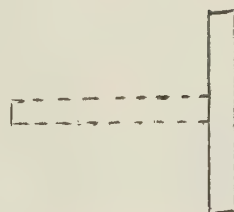
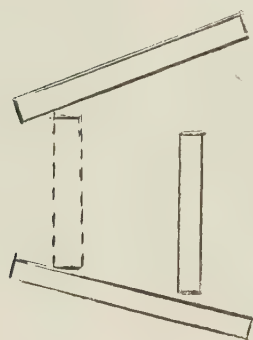


As for item 19

24. 9.5C
10B
9.5 Or



As for item 19











* B = Black
** Or = Orange

A P P E N D I X C

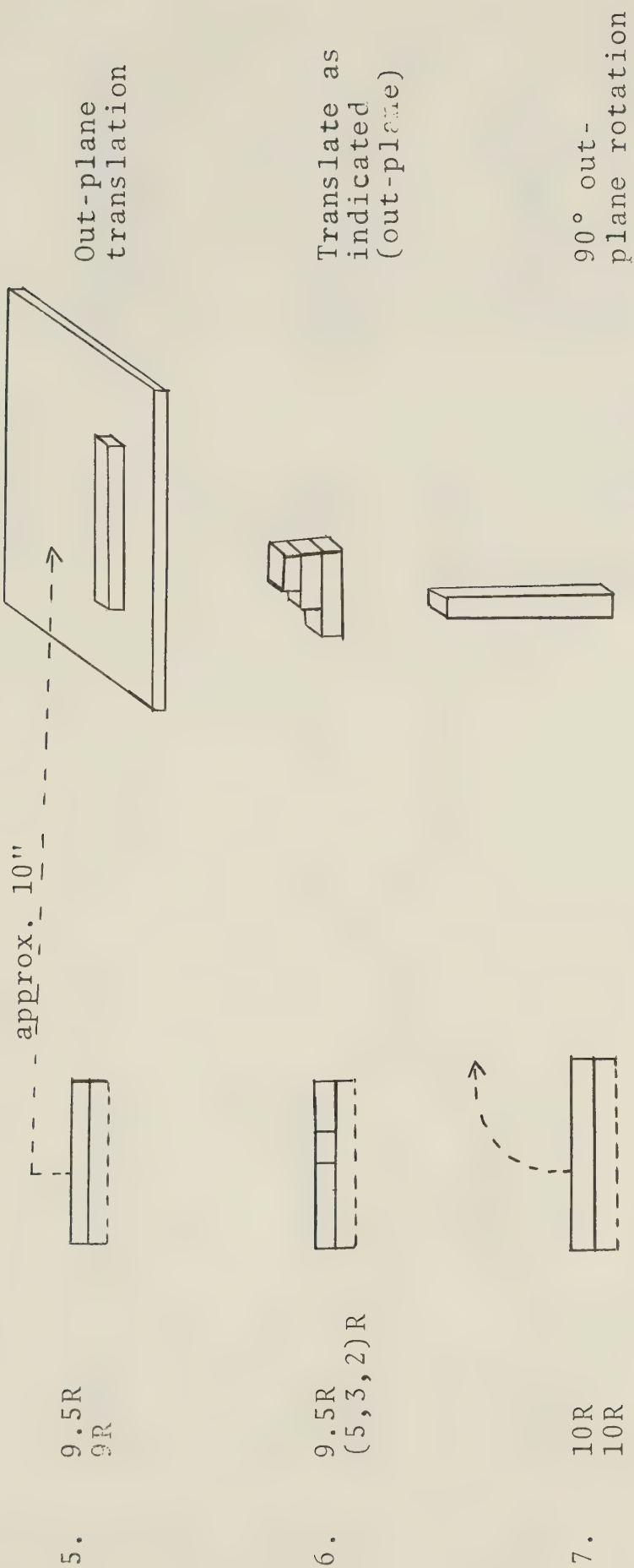
VERBAL CONSERVATION TEST

A VERBAL TRANSFORMATION TEST OF THE CONSERVATION OF LENGTH

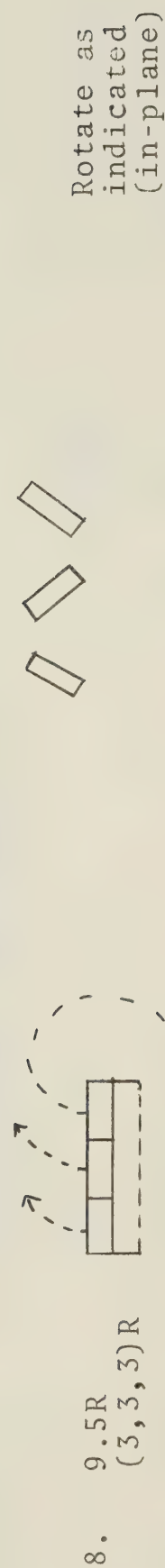
Test Item	Materials	Original Configuration	Transformed Configuration	Transformation Remarks
SUBTEST I. TRANSLATIONS:				
1.	10R* 10R			Out-plane translation
2.	9R (3, 3, 3)R			Translate as indicated (in-plane)
3.	9R (3, 3, 3)R			Translate as indicated (out-plane)
4.	9.5R 9R			Translate as indicated (out-plane)

* R = Rods

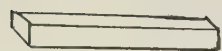
** Dotted lines indicate rods which were not transformed.



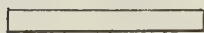
SUBTEST II. ROTATIONS:



90° out-plane
rotation



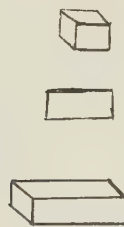
90° in-plane
rotation



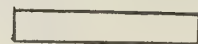
90° in-plane
rotations



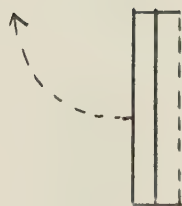
90° out-plane
rotation



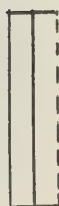
90° in-plane
rotation followed
by subtraction
of .5 cm. rod.



9. 9.5R
9R



10. 9.5R
10R



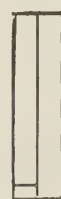
11. 9.5R
(5, 3, 2)R



12. 9.5R
(5, 3, 2)R




13. 9.5R
(.5, 9)R

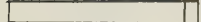


SUBTEST III. NON-CONSERVATION TRANSFORMATIONS:


14. 9.5R
(.5,9.5)R




90° in-plane
rotation followed
by addition of .5
cm. rod.




15. 10R
(5,2,3,
3.5)R
3.5 is white




Align 3 and 3.5
rods in identity
position. Then
replace 3 rod with
3.5 rod.



16. 10R
(5,3,2)R
(4.5,3,2)R
5 is black
4.5 is white




Align (5,3,2) and
(4.5,3,2) in
identity position.
Then remove (5,3,2).




17. 9.5R
9.5R
plasticene




Pound one end of
the plasticene in
about .5 cm.



18. 9.5R
9.5R

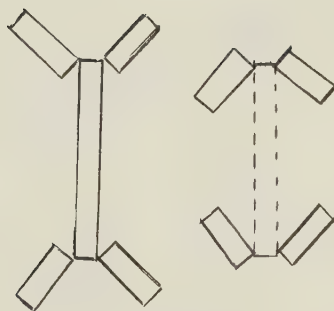


Roll the plasticene
under the palm so
that it gets slightly
longer.



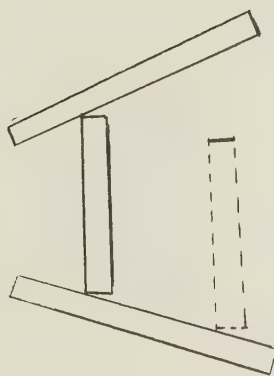
SUBTEST IV. ILLUSIONS:

19. 10B
100r



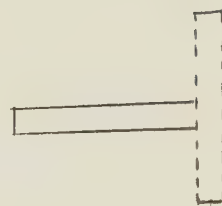
Place the rods into
the illusion forms
by any combinations
of translations
and rotations.

20. 10B
100r



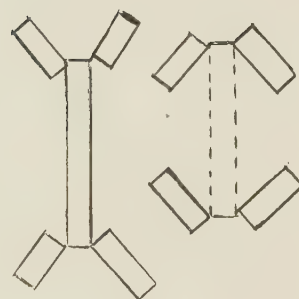
As for item 19

21. 10B
100r



As for item 19

22. 9.5B
100r

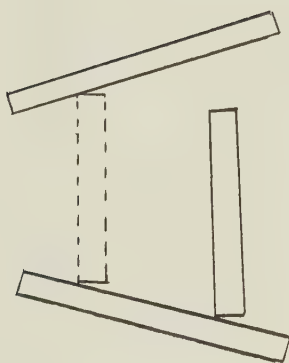


As for item 19

23. 10B *
9.50r



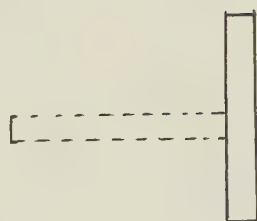
As for item 19



24. 10B
9.50r



As for item 19



* B= Black, Or= Orange
Dotted lines indicate orange rods.

A P P E N D I X D

INTERCORRELATION MATRIX FOR
ALL MAJOR VARIABLES

TABLE XVII
INTERCORRELATIONS AMONG MAJOR VARIABLES

Variable	1	2	3	4	5	6	7	8	9
1. Achievement	1.000								
2. Non-verbal Conservation	.316*	1.000							
3. Verbal Conservation	0.623**	--	1.000						
4. Intelligence	0.560**	0.272	0.555*	1.000					
5. Age	0.108	-0.028	-0.024	-0.260	1.000				
6. Sex	0.086	0.000	0.312*	0.072	-0.217	1.000			
7. SES	0.103	-0.085	0.088	0.235	-0.094	-0.112	1.000		
8. Conservers on Verbal Test	0.564**	--	0.814**	0.166	0.100	0.325*	0.202	1.000	
9. Conservers on Non-verbal Test	0.227	0.661**	--	0.114	-0.245	0.136	0.092	--	1.000

* Significant at .05 level.

** Significant at .001 level.

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